نظریه بازیها
Game Theory

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Extensive form

Material

- Intermediate microeconomics : a modern approach, varian : 8th Edition
  - Chapter 27

Market

- A monopoly is an industry consisting a single firm.
- A duopoly is an industry consisting of two firms.
- An oligopoly is an industry consisting of a few firms. Particularly, each firm’s own price or output decisions affect its competitors’ profits.
- How do we analyze markets in which the supplying industry is oligopolistic?

Basic model:Cournot

- Consider the duopolistic case of two firms supplying the same product.
  - duopoly: 2 firms (no other firms can enter)
  - firms sell identical products
  - market that lasts only 1 period (product or service cannot be stored and sold later)

Quantity Competition

- Consider the duopolistic case of two firms supplying the same product.
- Assume that firms compete by choosing output levels.
- If firm 1 produces y_1 units and firm 2 produces y_2 units then total quantity supplied is $y_1 + y_2$. The market price will be $p(y_1 + y_2)$.
- The firms' total cost functions are $c_1(y_1)$ and $c_2(y_2)$.
- Suppose firm 1 takes firm 2's output level choice y_2 as given. Then firm 1 sees its profit function as

$$\Pi_1(y_1; y_2) = p(y_1 + y_2)y_1 - c_1(y_1).$$

- Given y_2, what output level y_1 maximizes firm 1's profit?

Quantity Competition

- Generally, given firm 2's chosen output level y_2, firm 1's profit function is

$$\Pi_1(y_1; y_2) = p(y_1 + y_2)y_1 - c_1(y_1)$$

- and the profit-maximizing value of y_1 solves

$$\frac{\partial \Pi_1}{\partial y_1} = p(y_1 + y_2) + y_1 \frac{\partial p(y_1 + y_2)}{\partial y_1} - c_1{}'(y_1) = 0.$$

- The solution, $y_1 = R_1(y_2)$, is firm 1's Cournot-Nash reaction to y_2.

Quantity Competition

- In a similar way, firm 2 sees its profit function as

$$\Pi_2(y_1; y_2) = p(y_1 + y_2)y_2 - c_2(y_2).$$

- Suppose that the market inverse demand function is

$$p(y_T) = a - (y_1 + y_2) = 60 - y_T$$

- and that the firms' total cost functions are
- $c_1(y_1) = y_1^2$ and $c_2(y_2) = 15y_2 + y_2^2.$

Quantity Competition: An Example

- Then, for given y_2, firm 1's profit function is

$$\Pi(y_2; y_1) = (60 - y_1 - y_2)y_1 - y_1^2.$$

- So, given y_2, firm 1's profit-maximizing output level solve

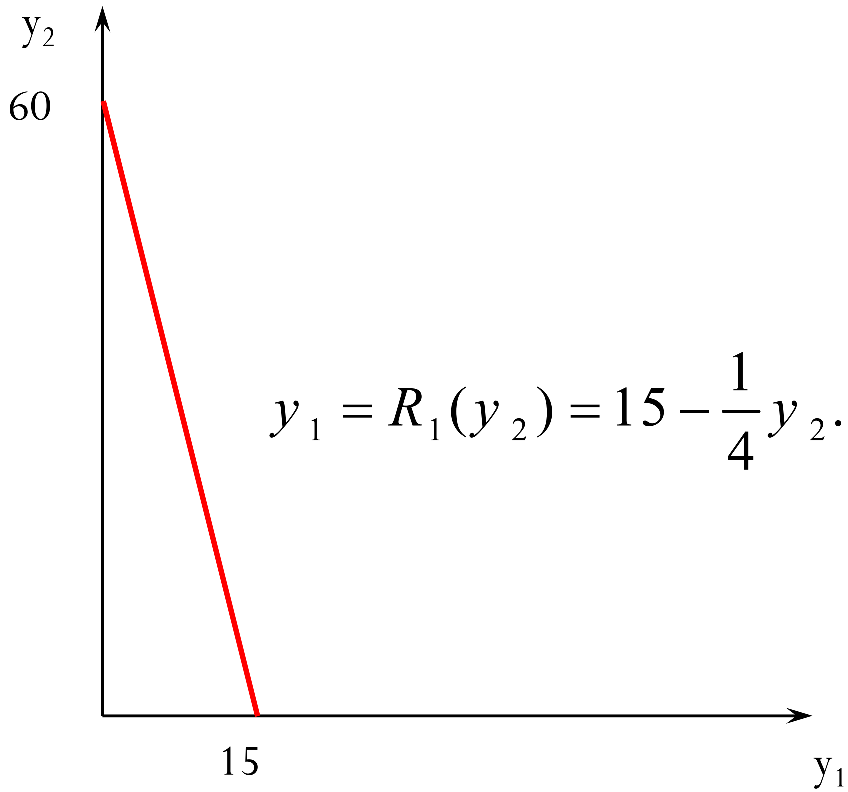
$$\frac{\partial \Pi}{\partial y_1} = 60 - 2y_1 - y_2 - 2y_1 = 0.$$

- I.e. firm 1's best response to y_2 is

$$y_1 = R_1(y_2) = 15 - \frac{1}{4}y_2.$$

Quantity Competition: An Example

- Firm 1's "reaction curve"



$$y_1 = R_1(y_2) = 15 - \frac{1}{4}y_2.$$

Quantity Competition: An Example

- Then, for given y_2, firm 2's profit function is

$$\Pi(y_1; y_2) = (60 - y_1 - y_2)y_2 - 15y_2 - y_2^2.$$

- So, given y_2, firm 2's profit-maximizing output level solve

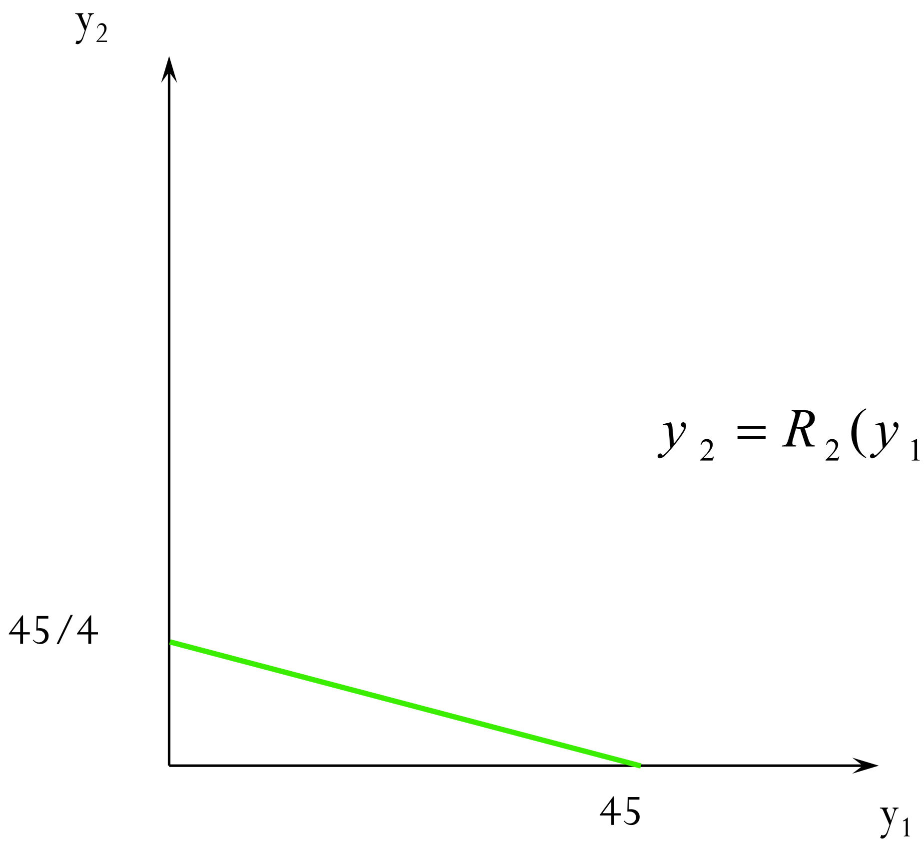
$$\frac{\partial \Pi}{\partial y_2} = 60 - y_1 - 2y_2 - 15 - 2y_2 = 0$$

- I.e. firm 2's best response to y_1 is

$$y_2 = R_2(y_1) = \frac{45 - y_1}{4}$$

Quantity Competition: An Example

- Firm 2's "reaction curve"



$$y_2 = R_2(y_1) = \frac{45 - y_1}{4}.$$

Quantity Competition: An Example

- $y_1^* = R_1(y_2^*) = 15 - \frac{1}{4} y_2^*$ and $y_2^* = R_2(y_1^*) = \frac{45 - y_1^*}{4}.$
- Substitute for y_2^* to get

$$y_1^* = 15 - \frac{1}{4}\left(\frac{45 - y_1^*}{4}\right) \Rightarrow y_1^* = 13$$

- Hence

$$y_2^* = \frac{45 - 13}{4} = 8.$$

- So the Cournot-Nash equilibrium is

$$(y_1^*, y_2^*) = (13, 8).$$

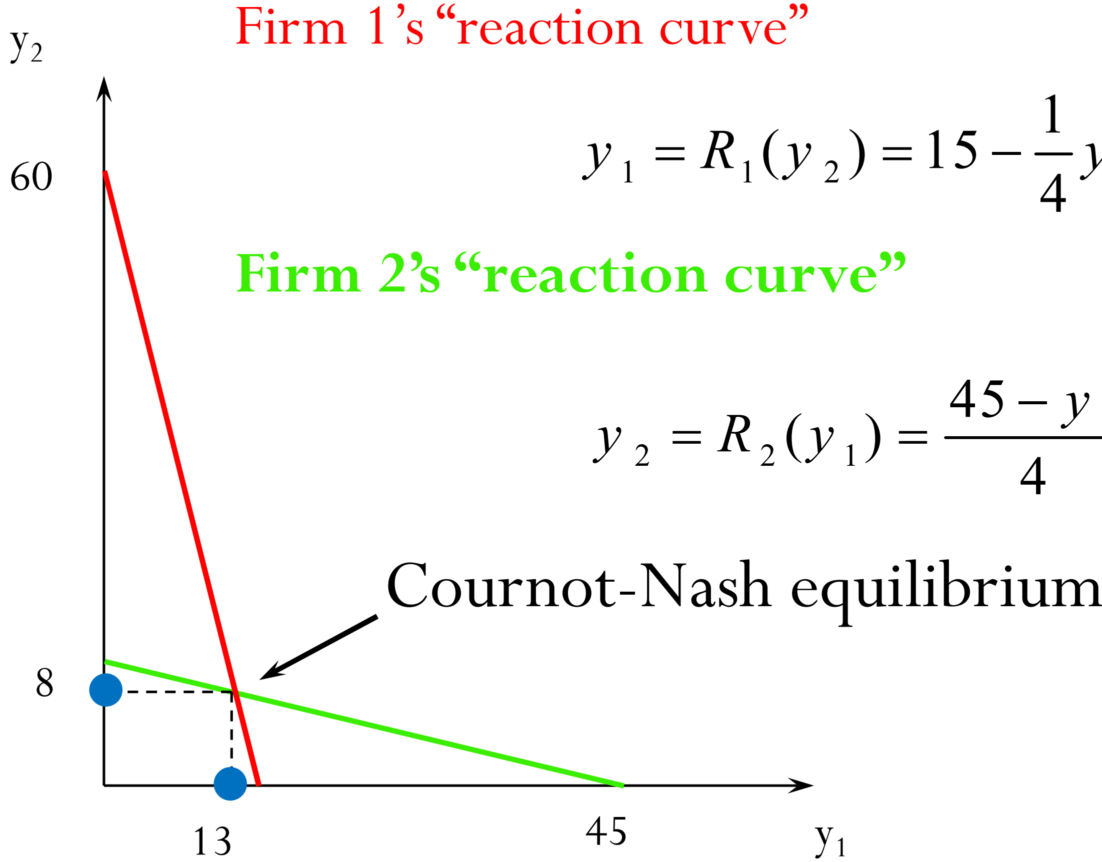
Quantity Competition: An Example

Firm 1's "reaction curve"

$$y_1 = R_1(y_2) = 15 - \frac{1}{4} y_2.$$

Firm 2's "reaction curve"

$$y_2 = R_2(y_1) = \frac{45 - y_1}{4}.$$



Quantity Competition

- Generally, given firm 2's chosen output level y_2, firm 1's profit function is

$$\Pi_1(y_1; y_2) = p(y_1 + y_2)y_1 - c_1(y_1)$$

- and the profit-maximizing value of y_1 solves

$$\frac{\partial \Pi_1}{\partial y_1} = p(y_1 + y_2) + y_1 \frac{\partial p(y_1 + y_2)}{\partial y_1} - c_1'(y_1) = 0.$$

- The solution, $y_1 = R_1(y_2)$, is firm 1's Cournot-Nash reaction to y_2.

Quantity Competition

- Similarly, given firm 1's chosen output level y_1, firm 2's profit function is

$$\Pi_2(y_2; y_1) = p(y_1 + y_2)y_2 - c_2(y_2)$$

- and the profit-maximizing value of y_2 solves

$$\frac{\partial \Pi_2}{\partial y_2} = p(y_1 + y_2) + y_2 \frac{\partial p(y_1 + y_2)}{\partial y_2} - c_2'(y_2) = 0.$$

- The solution, $y_2 = R_2(y_1)$, is firm 2's Cournot-Nash reaction to y_1.

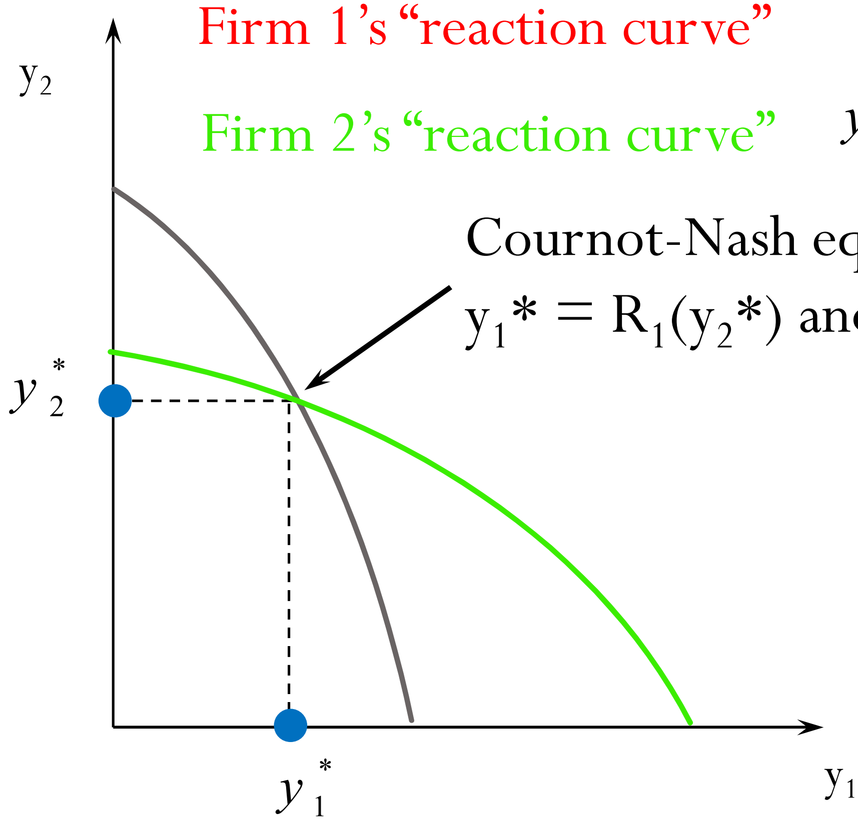
Quantity Competition

Firm 1's "reaction curve" $y_1 = R_1(y_2)$.

Firm 2's "reaction curve" $y_2 = R_2(y_1)$.

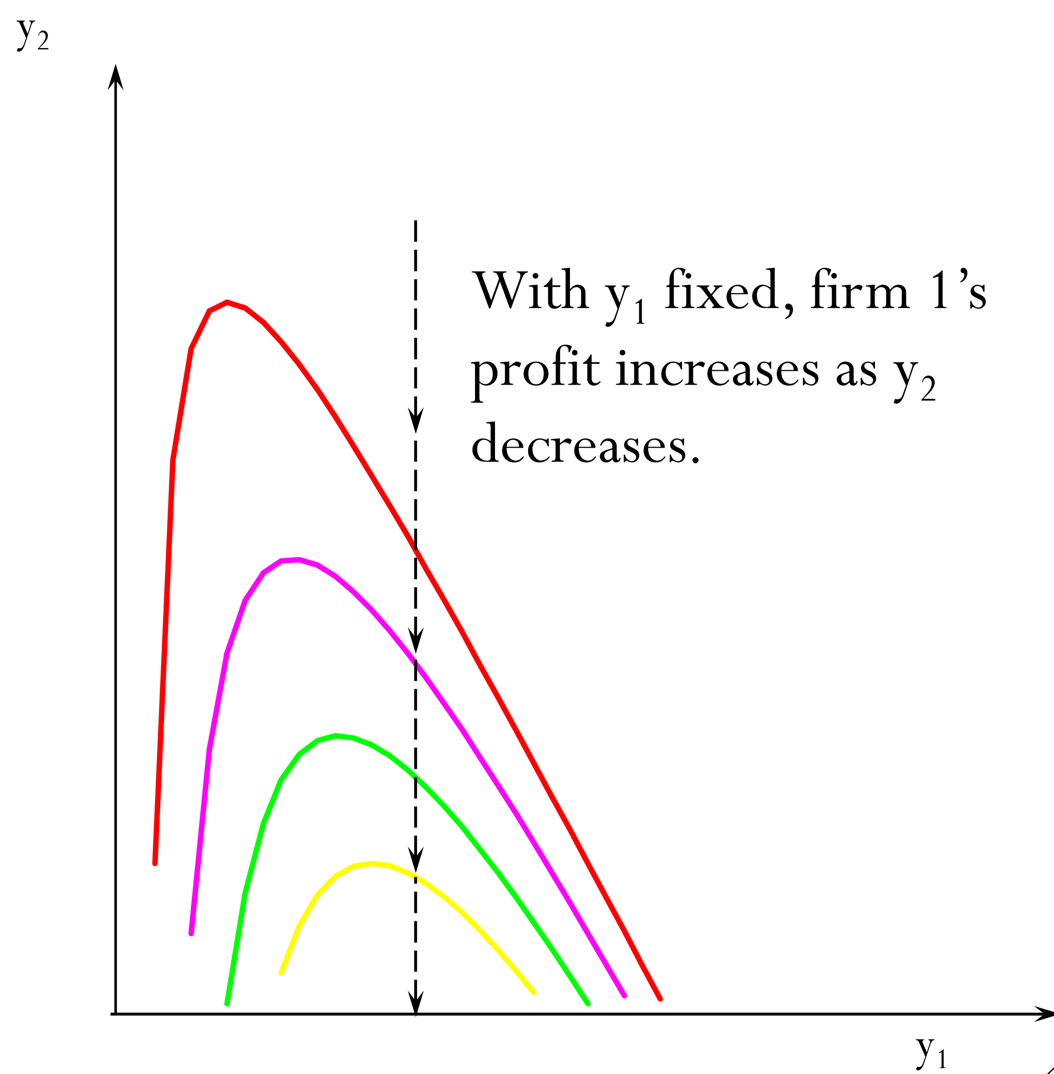
Cournot-Nash equilibrium

$y_1^* = R_1(y_2^*)$ and $y_2^* = R_2(y_1^*)$

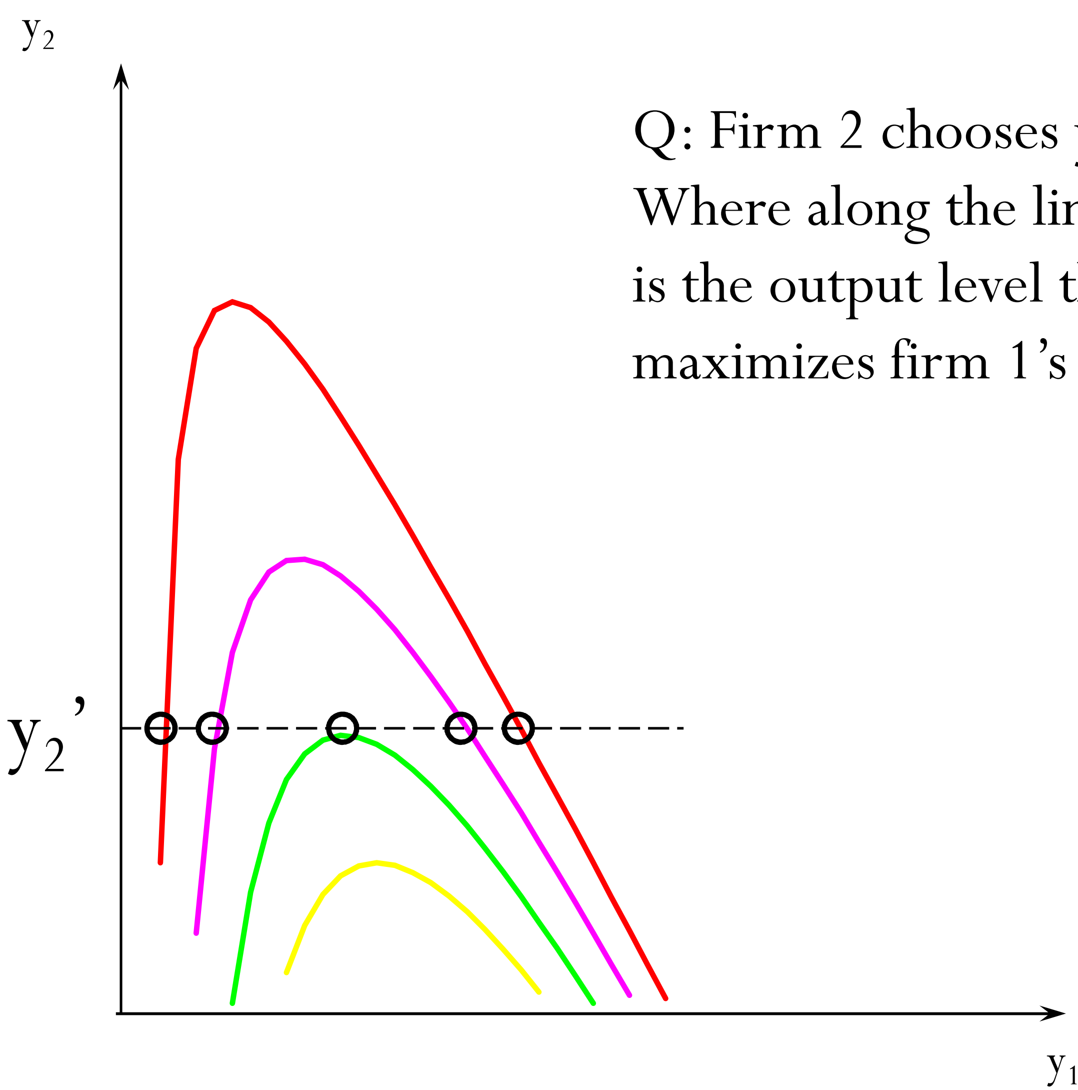


Iso-Profit Curves

- For firm 1, an iso-profit curve contains all the output pairs (y_1, y_2) giving firm 1 the same profit level Π_1.
- What do iso-profit curves look like?

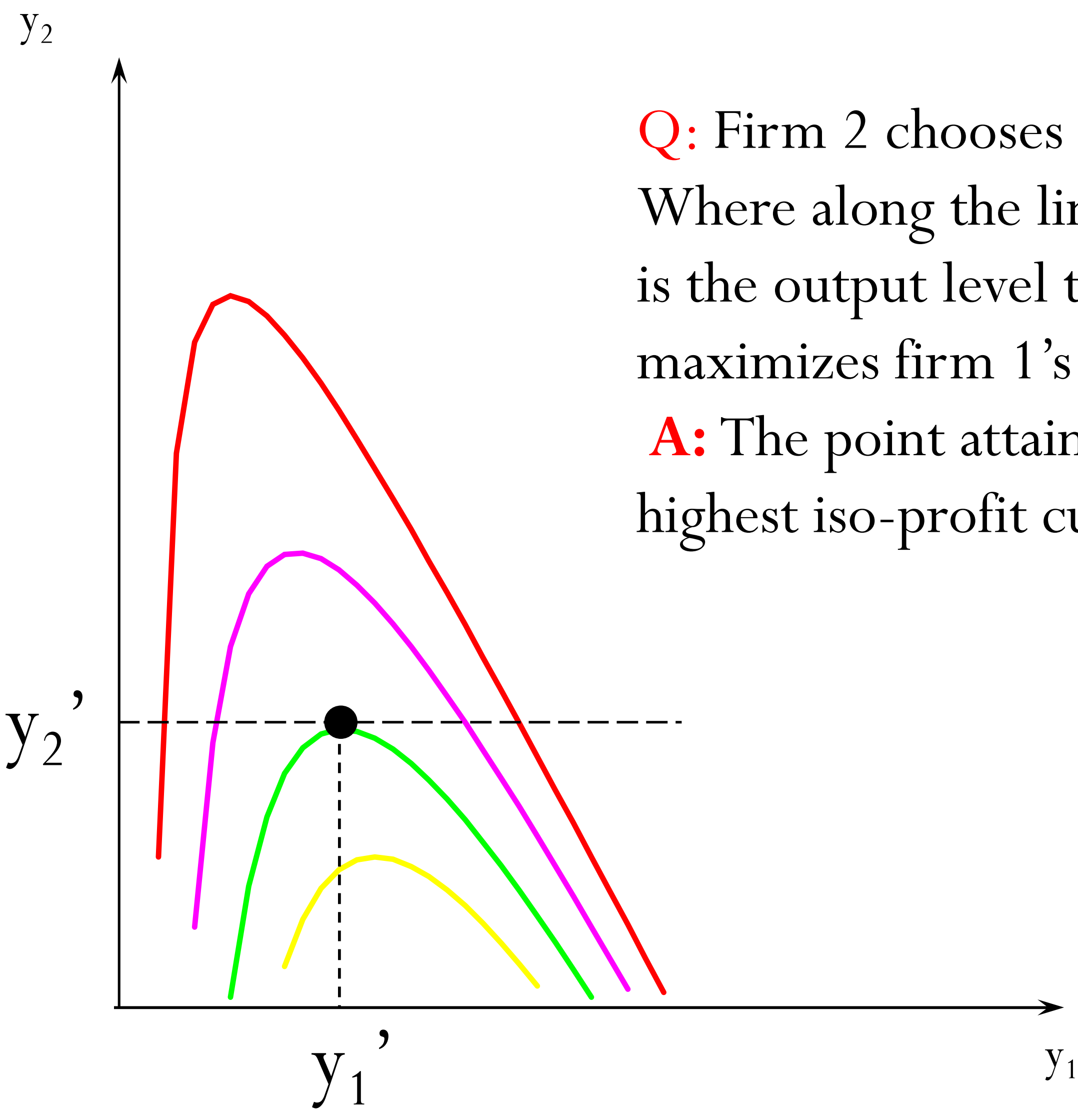


Iso-Profit Curves for Firm 1



Q: Firm 2 chooses $y_2 = y_2'$.
Where along the line $y_2 = y_2'$ is the output level that maximizes firm 1's profit?

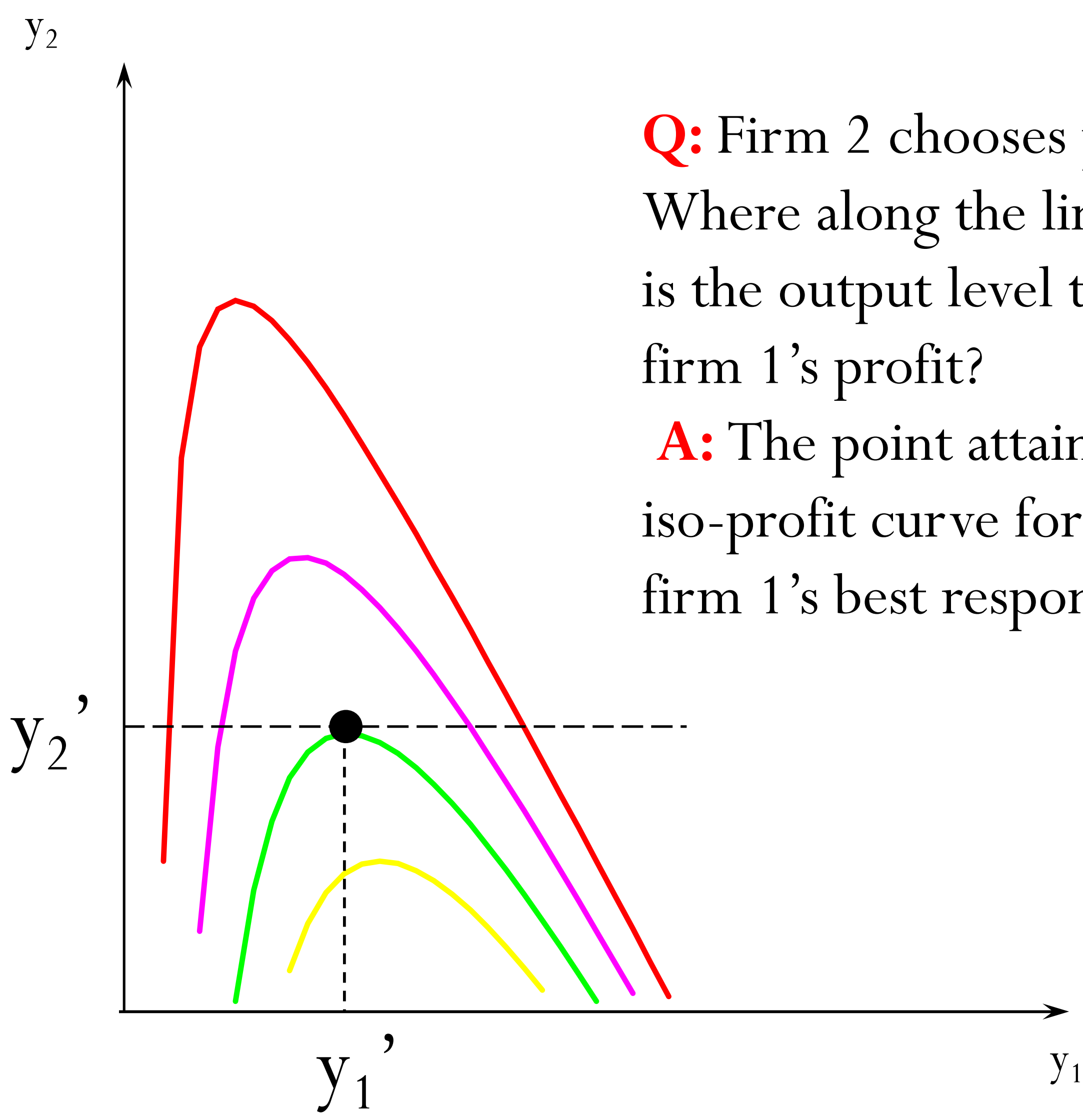
Iso-Profit Curves for Firm 1



Q: Firm 2 chooses $y_2 = y_2'$. Where along the line $y_2 = y_2'$ is the output level that maximizes firm 1's profit?

A: The point attaining the highest iso-profit curve for firm 1.

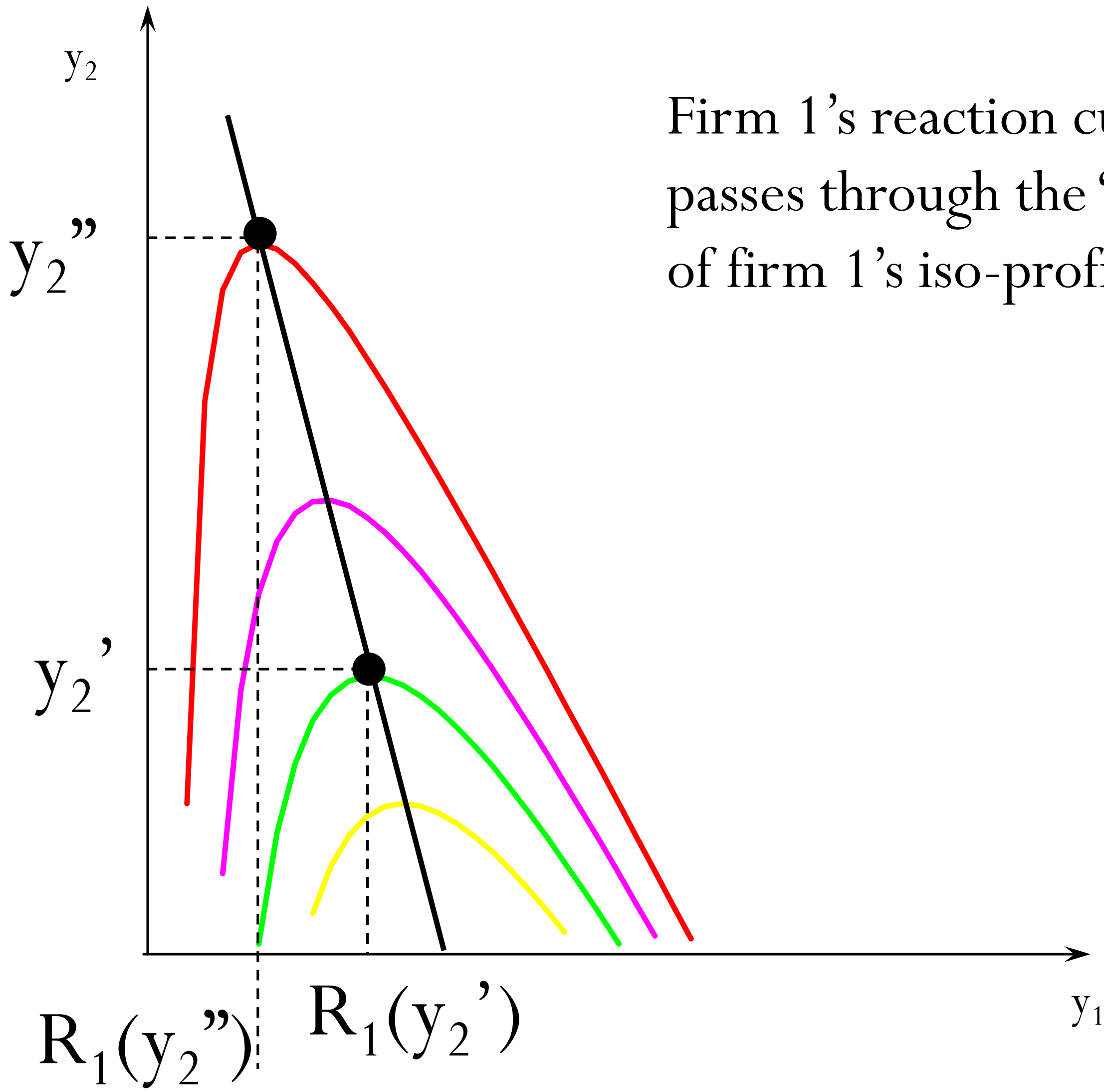
Iso-Profit Curves for Firm 1



Q: Firm 2 chooses $y_2 = y_2'$.
Where along the line $y_2 = y_2'$ is the output level that maximizes firm 1's profit?

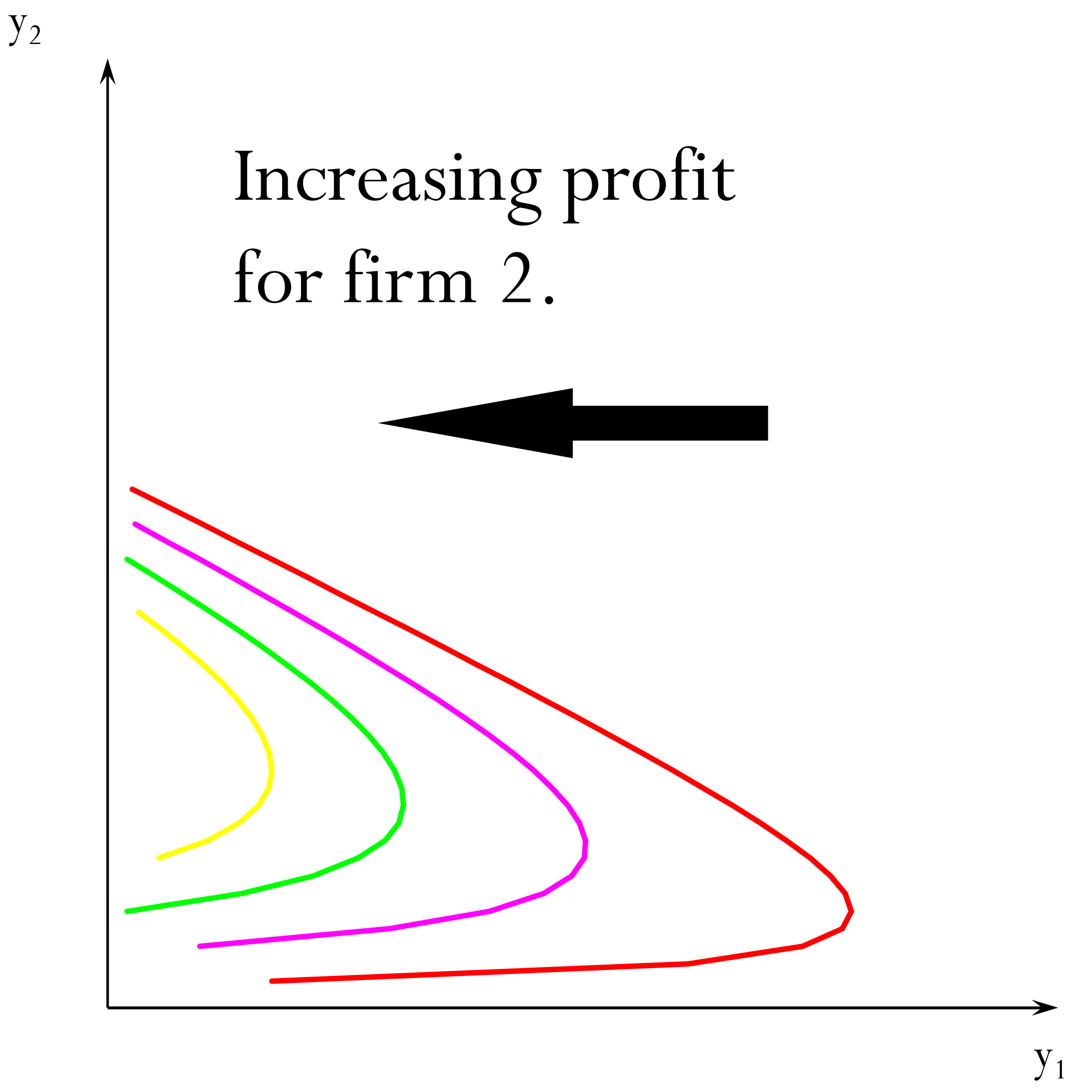
A: The point attaining the highest iso-profit curve for firm 1. y_1' is firm 1's best response to $y_2 = y_2'$.

Iso-Profit Curves for Firm 1

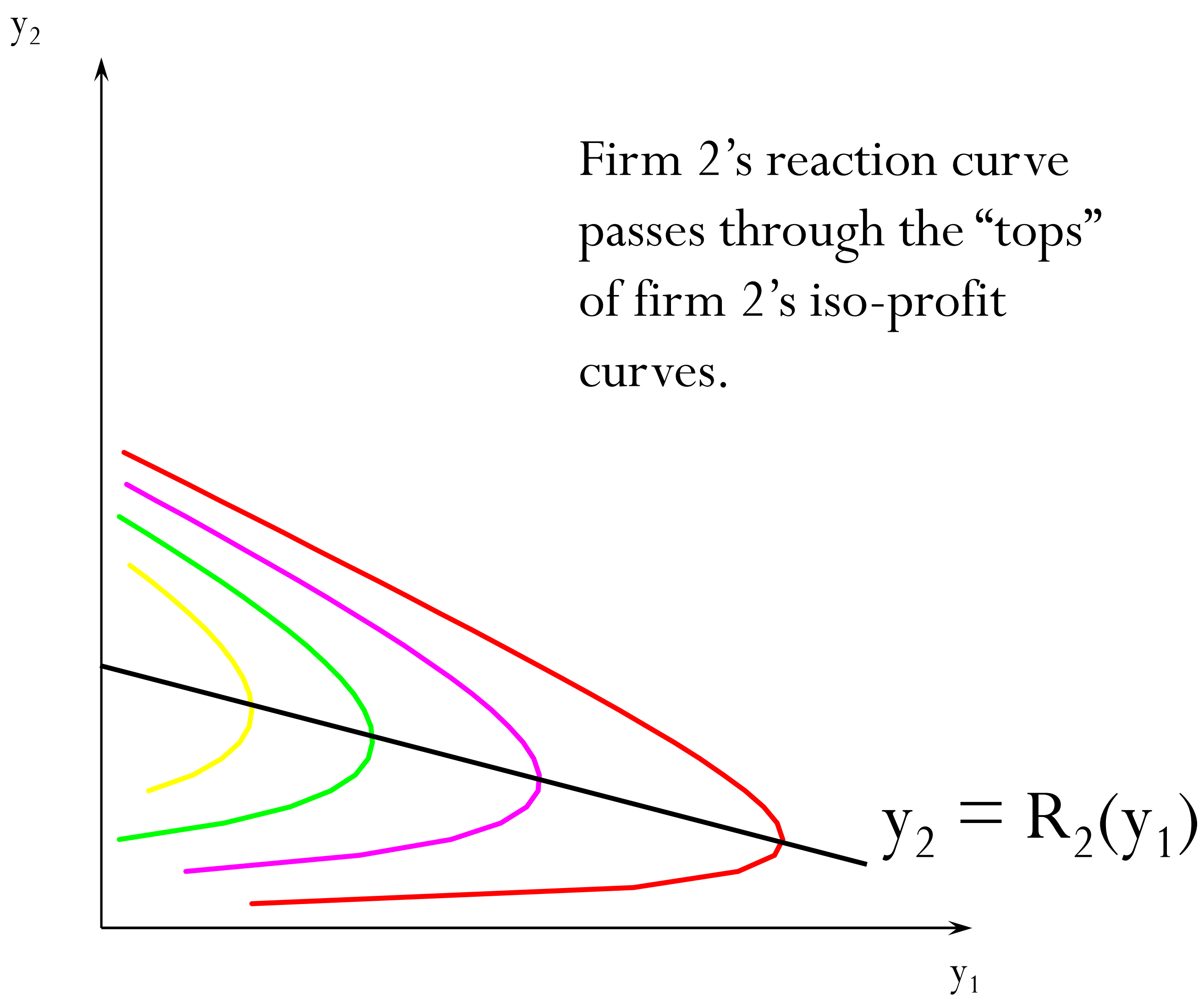


Firm 1's reaction curve passes through the "tops" of firm 1's iso-profit curves.

Iso-Profit Curves for Firm 2

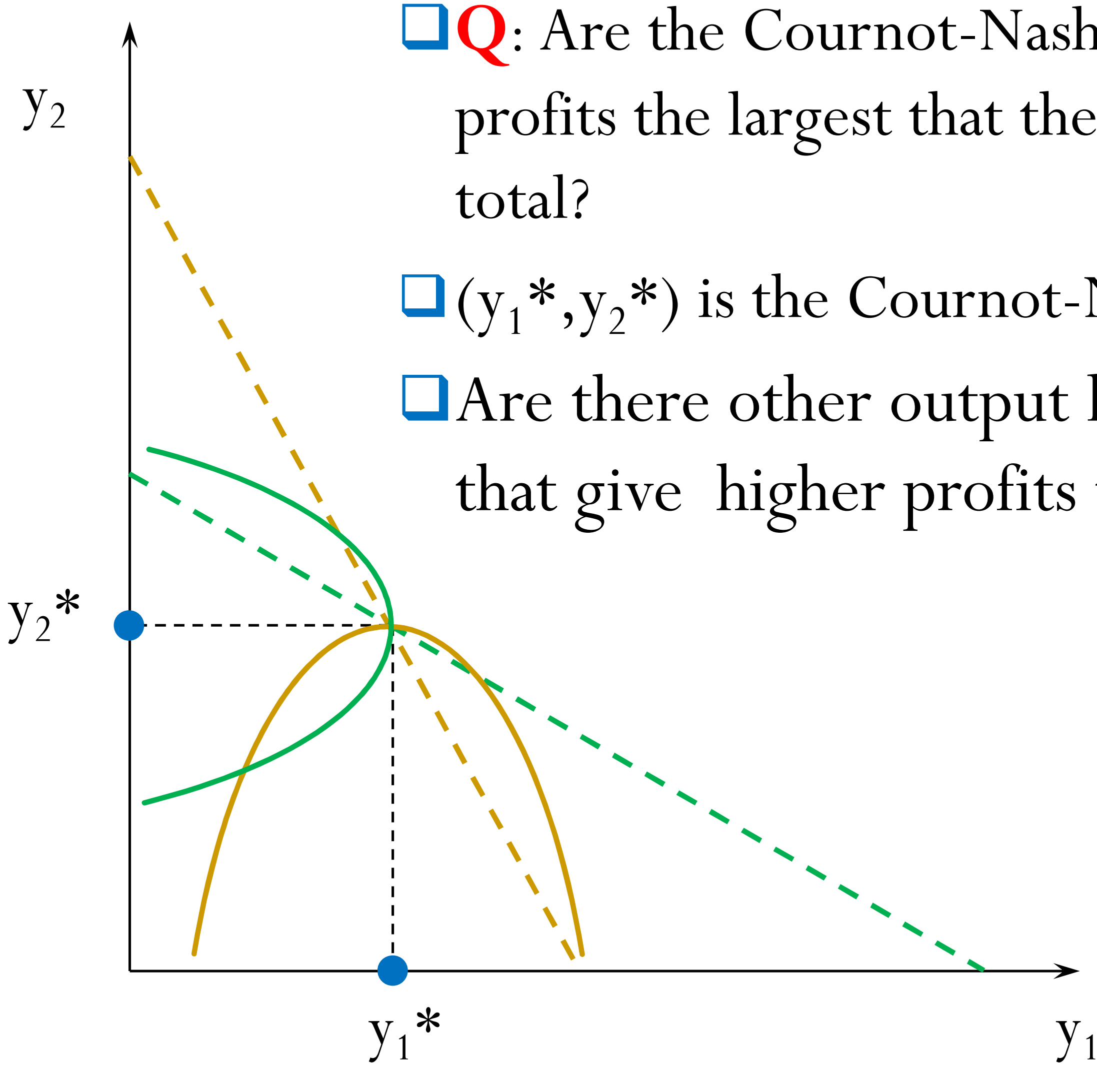


Iso-Profit Curves for Firm 2



Firm 2's reaction curve passes through the "tops" of firm 2's iso-profit curves.

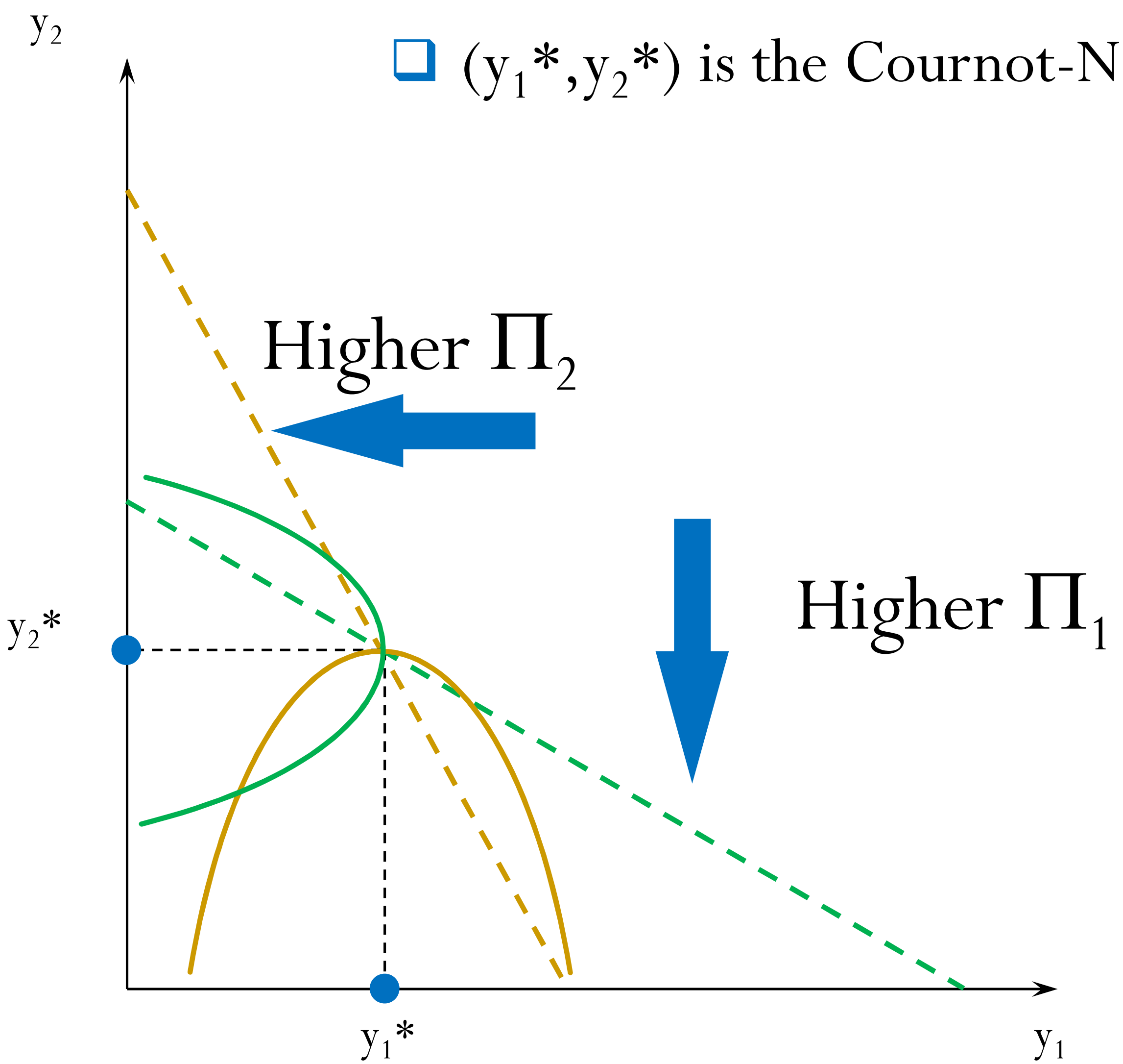
Collusion



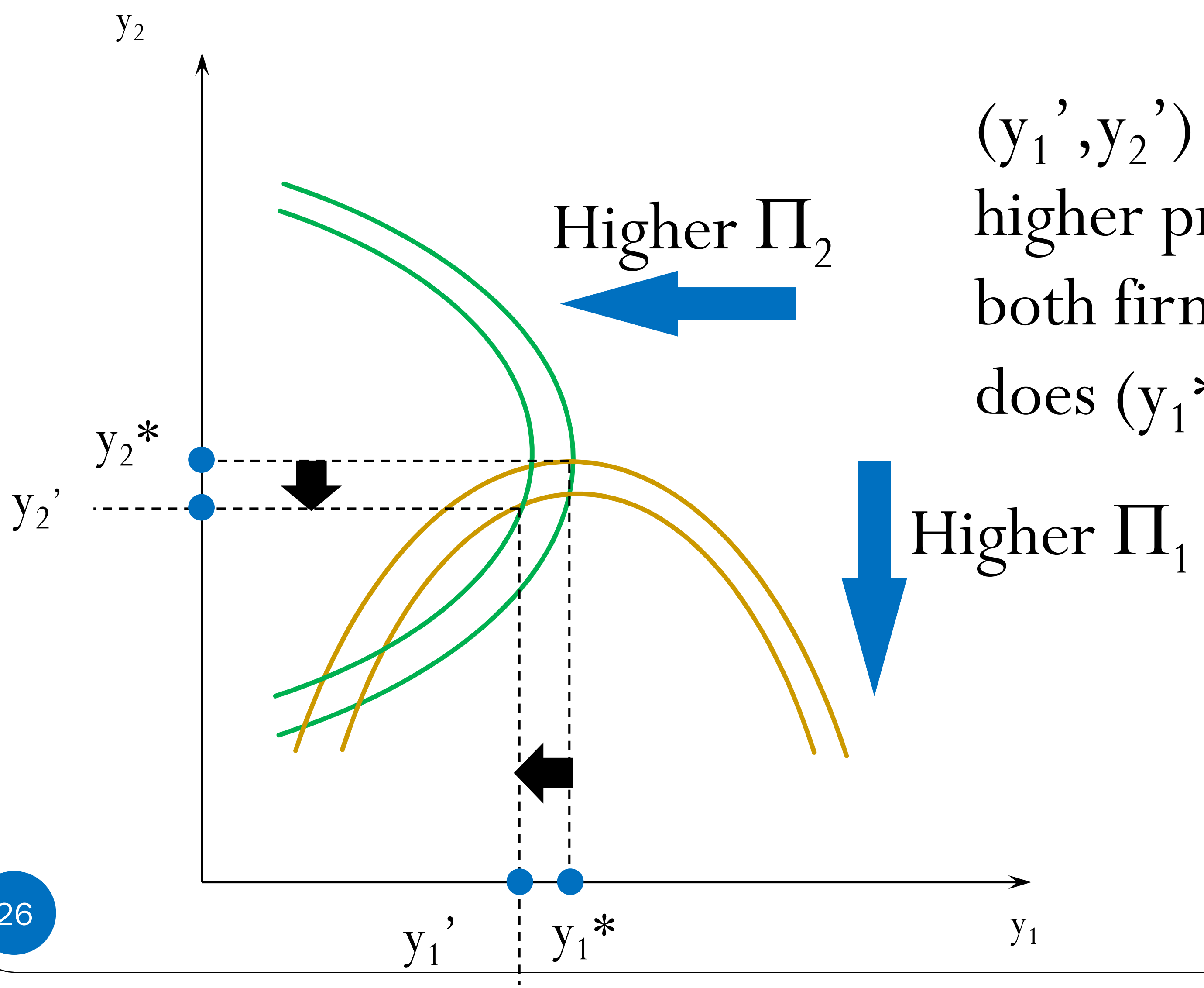
- **Q**: Are the Cournot-Nash equilibrium profits the largest that the firms can earn in total?
- (y_1^*, y_2^*) is the Cournot-Nash equilibrium.
- Are there other output level pairs (y_1, y_2) that give higher profits to both firms?

Collusion

- ❑ (y_1^*, y_2^*) is the Cournot-Nash equilibrium.



Collusion



(y_1',y_2') earns higher profits for both firms than does (y_1*,y_2*).

Collusion

- So there are profit incentives for both firms to “cooperate” by lowering their output levels.
- This is **collusion.**
- Firms that collude are said to have formed a **cartel.**
- **If firms form a cartel, how should they do it?**

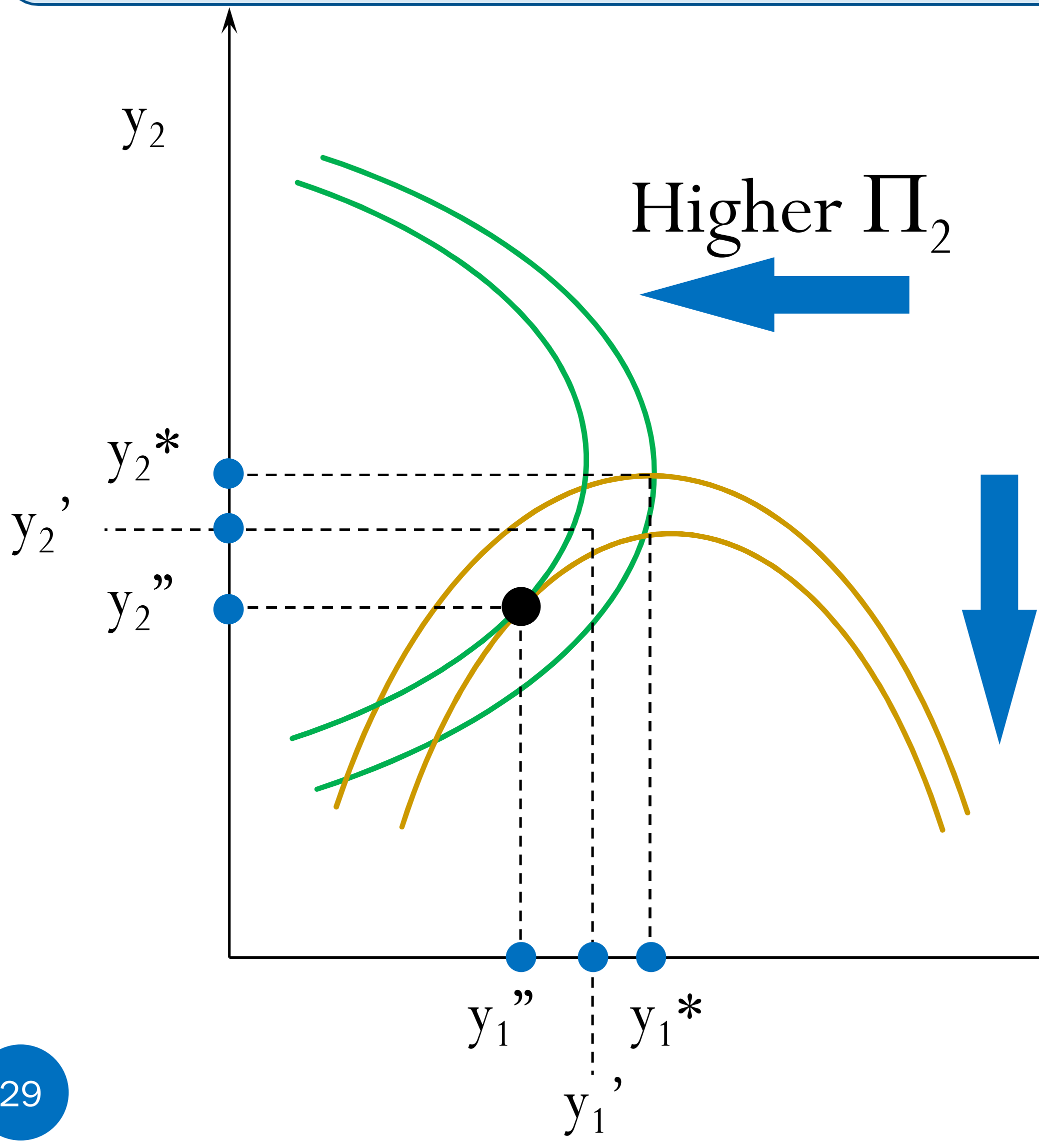
Collusion

- Suppose the two firms want to **maximize** their total profit and divide it between them. Their goal is to **choose cooperatively output** levels y_1 and y_2 that maximize

$$\Pi^m(y_1, y_2) = p(y_1 + y_2)(y_1 + y_2) - c_1(y_1) - c_2(y_2).$$

- The firms cannot do worse by colluding since they can cooperatively choose their Cournot-Nash equilibrium output levels and so earn their Cournot-Nash equilibrium profits. So collusion must provide **profits at least as large as** their Cournot-Nash equilibrium profits.

Collusion

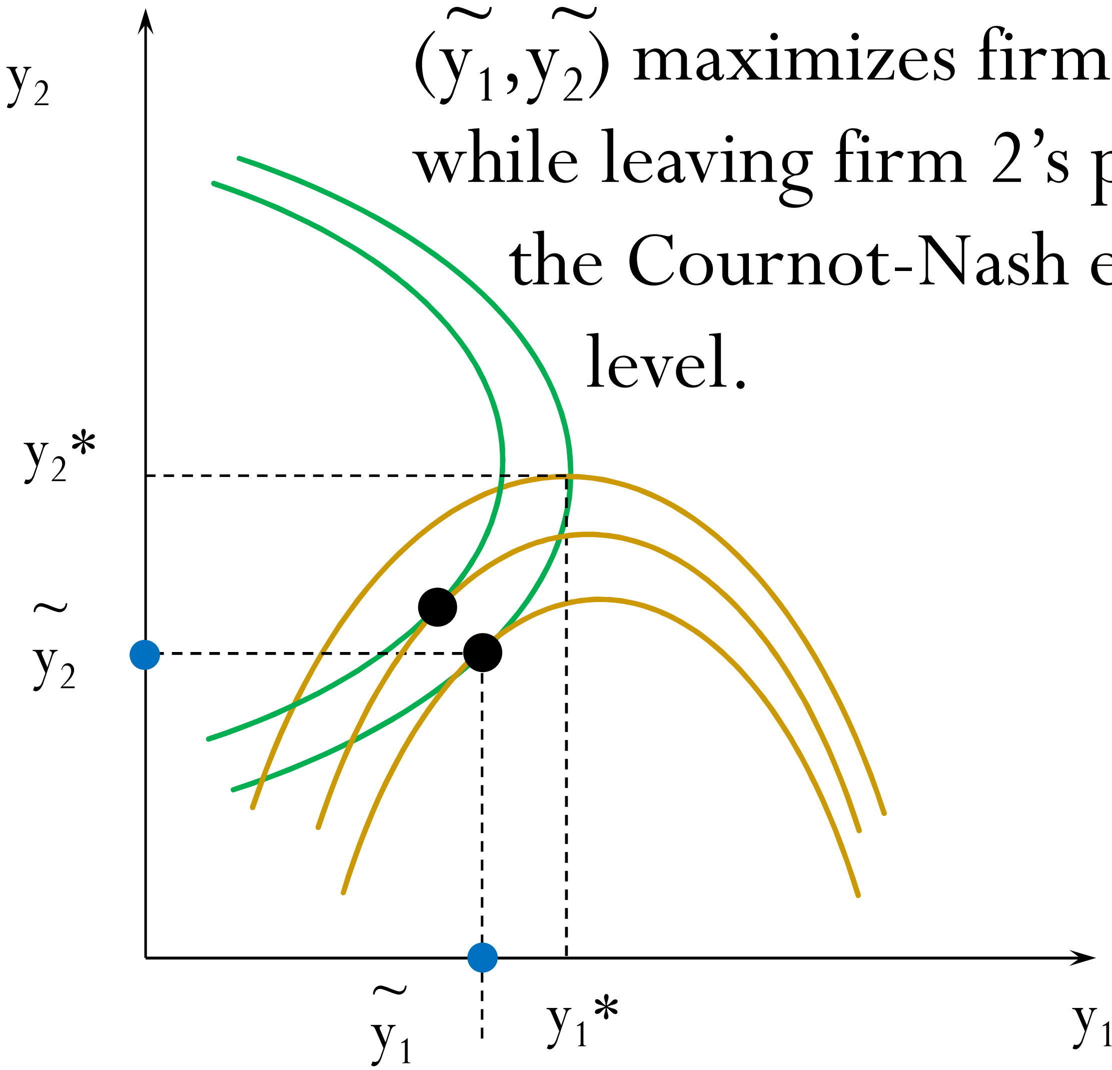


(y_1', y_2') earns higher profits for both firms than does (y_1^*, y_2^*).

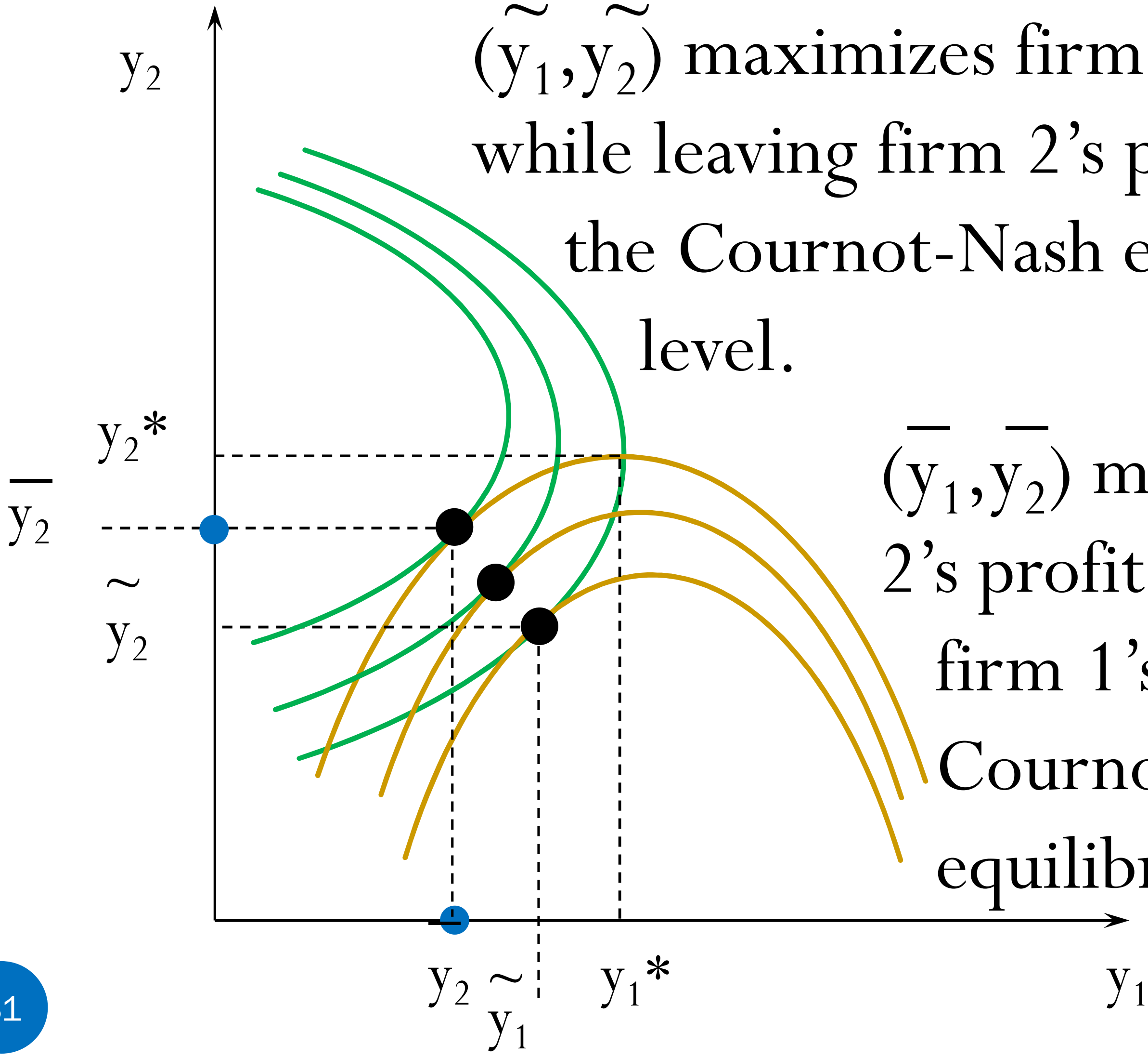
(y_1'', y_2'') earns still higher profits for both firms.

Collusion

$(\tilde{y}_1, \tilde{y}_2)$ maximizes firm 1's profit while leaving firm 2's profit at the Cournot-Nash equilibrium level.



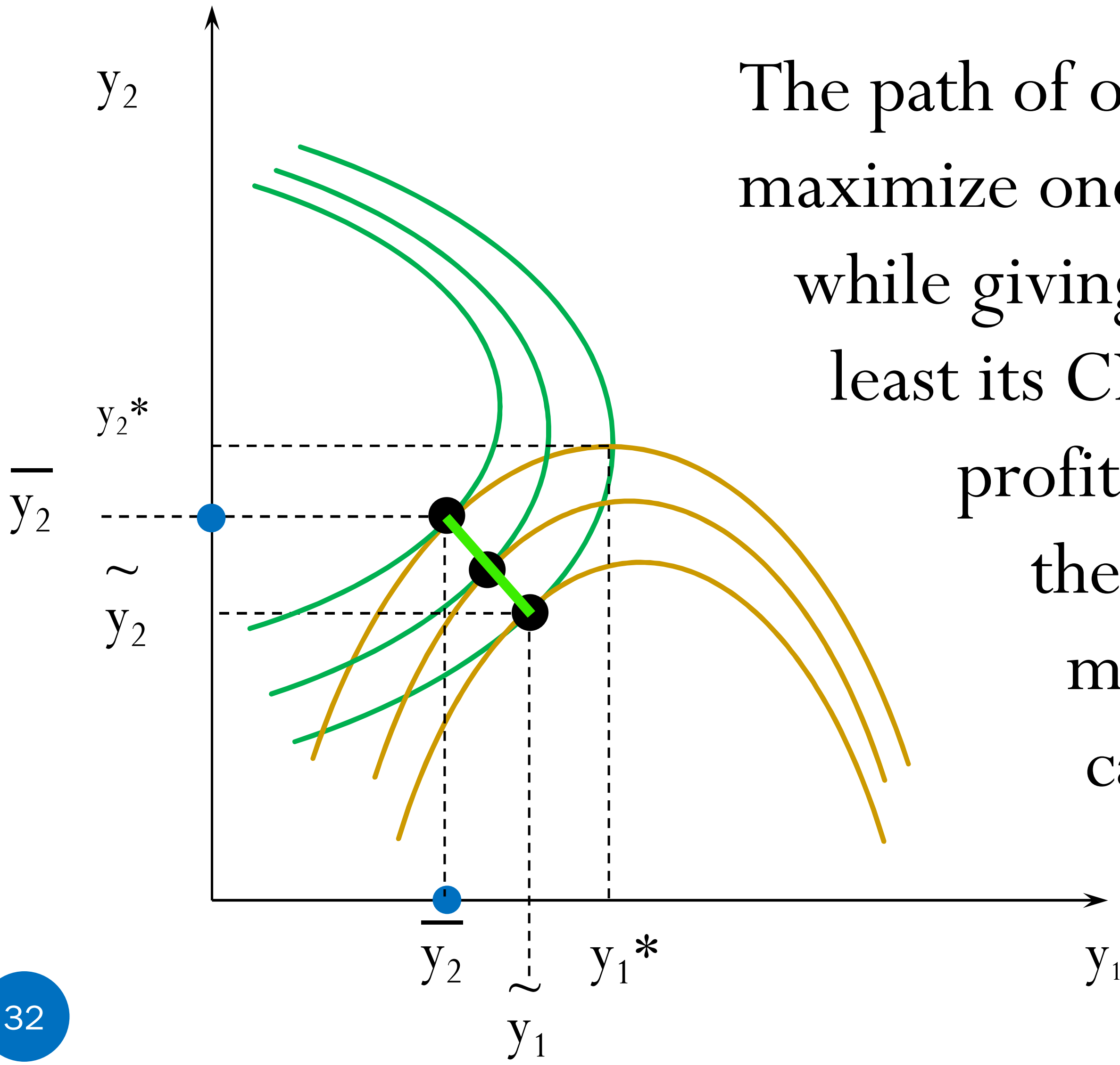
Collusion



$(\tilde{y}_1, \tilde{y}_2)$ maximizes firm 1's profit while leaving firm 2's profit at the Cournot-Nash equilibrium level.

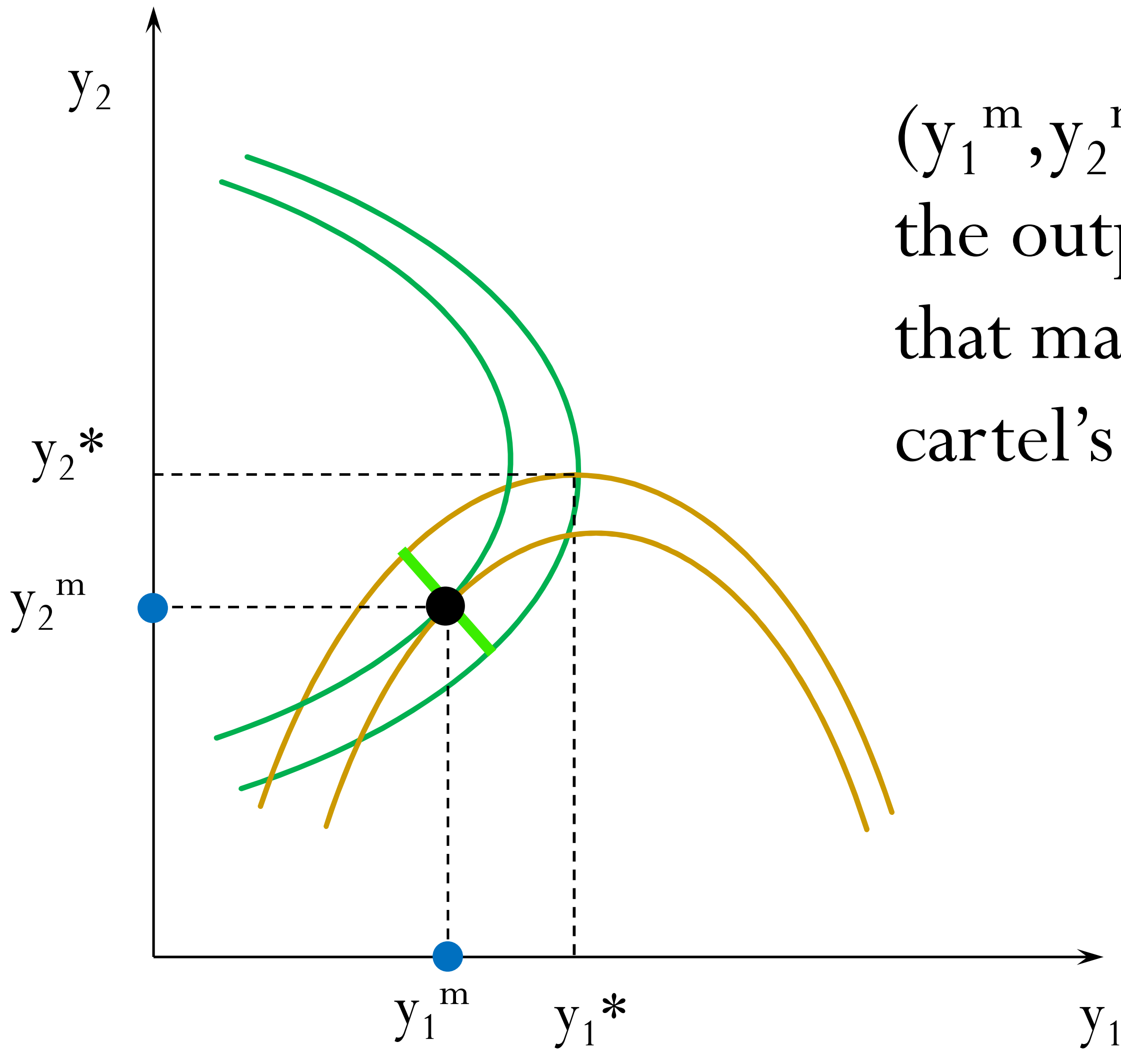
$(\bar{y}_1, \bar{y}_2)$ maximizes firm 2's profit while leaving firm 1's profit at the Cournot-Nash equilibrium level.

Collusion



The path of output pairs that maximize one firm's profit while giving the other firm at least its CN equilibrium profit. One of these output pairs must maximize the cartel's joint profit.

Collusion



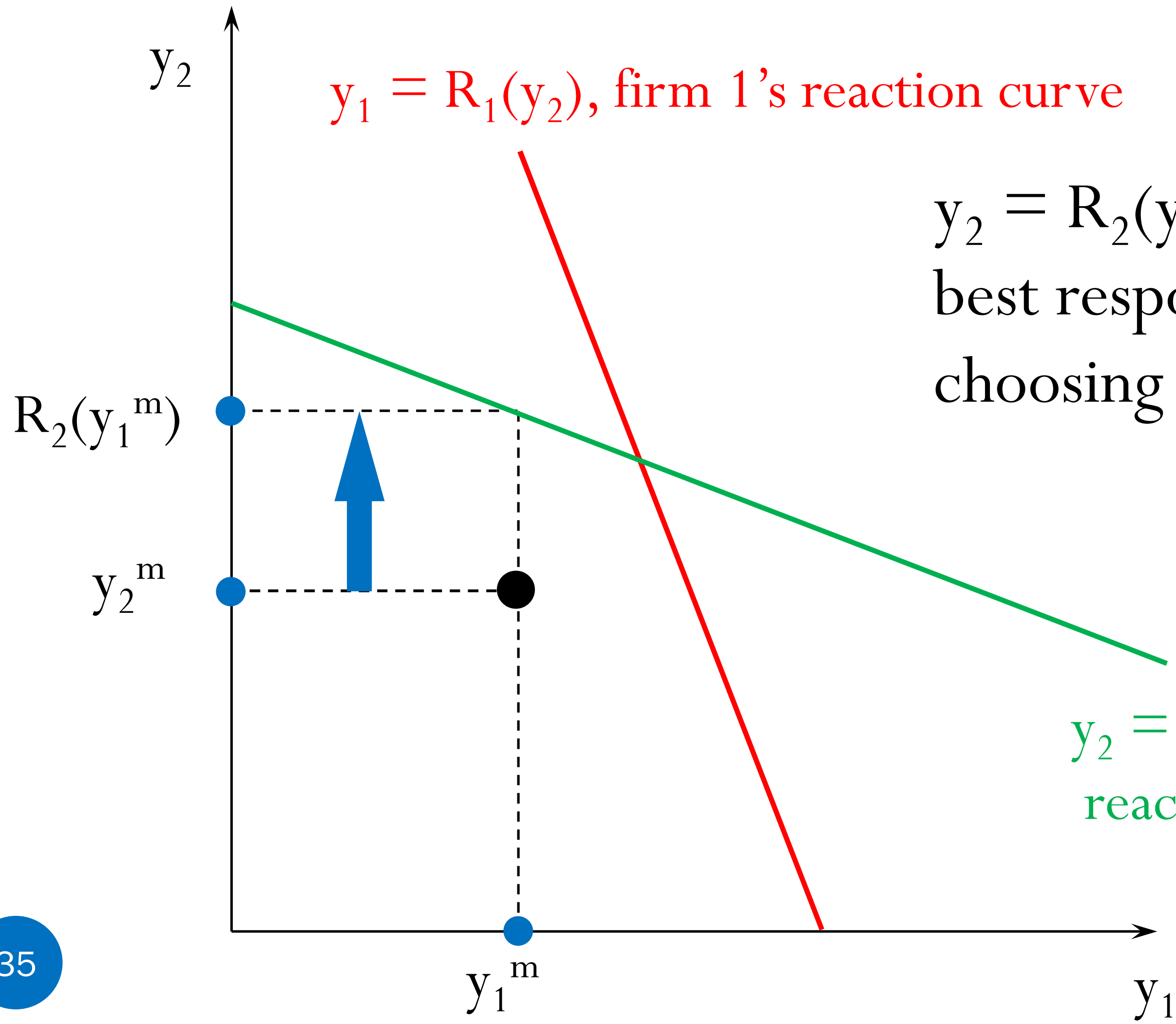
(y_1^m, y_2^m) denotes the output levels that maximize the cartel's total profit.

Collusion

- Is such a cartel stable?
- Does one firm have an **incentive to cheat** on the other?
- I.e. if firm 1 continues to produce y_1^m units, is it profit-maximizing for firm 2 to continue to produce y_2^m units?
- Firm 2's profit-maximizing response to $y_1 = y_1^m$ is

$y_2 = R_2(y_1^m)$.

Collusion

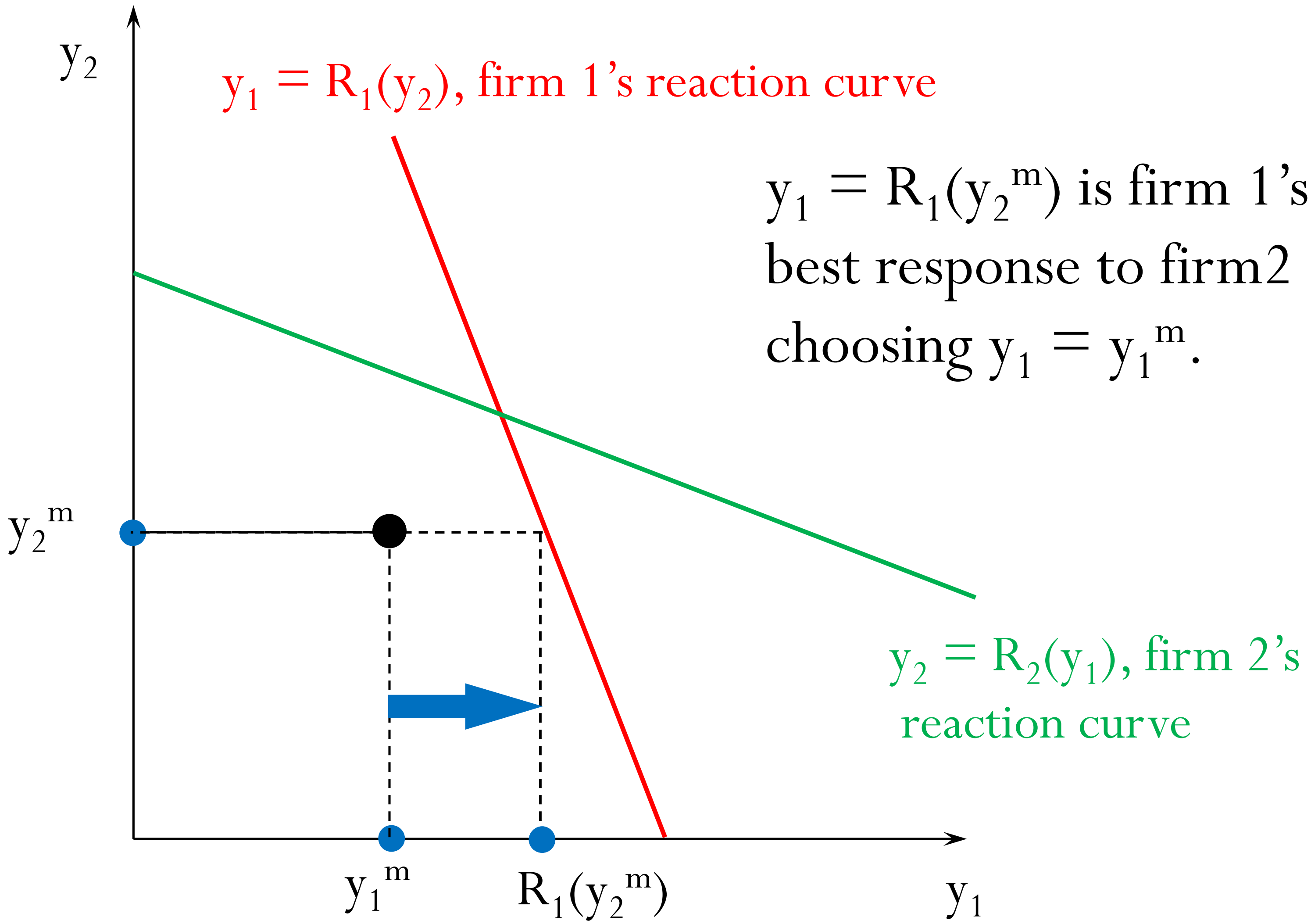


$y_2 = R_2(y_1^m)$ is firm 2's best response to firm1 choosing $y_1 = y_1^m$.

Collusion

- Firm 2's profit-maximizing response to $y_1 = y_1^m$ is $y_2 = R_2(y_1^m) > y_2^m$.
- Firm 2's profit increases if it cheats on firm 1 by increasing its output level from y_2^m to $R_2(y_1^m)$.
- Similarly, firm 1's profit increases if it cheats on firm 2 by increasing its output level from y_1^m to $R_1(y_2^m)$.

Collusion



$y_1 = R_1(y_2{}^m)$ is firm 1's best response to firm2 choosing $y_1 = y_1{}^m$.

Collusion

Why cartels fail?

- cartels fail if noncartel members can supply consumers with large quantities of goods (example: copper)
- each member of a cartel has an incentive to cheat on cartel agreement
- So a profit-seeking cartel in which firms cooperatively set their output levels is fundamentally unstable.
- E.g. OPEC's broken agreements.

Soft drinks 1986 merger proposals

- Coke, largest producer of carbonated soft drinks (38.6% of sales), tried to buy third largest, Dr Pepper (7.1%)
- Pepsi, second largest producer (27.4%), tried to acquire fourth largest firm, Seven-Up Co. (6.3%)
- had these proposed mergers taken place, Coke's market share would have risen to 45.7% and Pepsi's to 33.7%
- combined share would have risen from 66.0% to 79.4%

FTC intervenes

Federal Trade Commission (FTC) opposed mergers, arguing that merger

- would increase market shares of big firms
- make entry of new firms more difficult
- raise costs of other companies doing business in this market
- ease "collusion among participants in the relevant markets"

Outcome

- after Coke and Pepsi mergers blocked by FTC in 1986
  - Dr Pepper Co. sold for $416 million to investor group ($54 million less than Coke offered)
  - Seven-Up Co. sold for $240 million to another investment group ($140 million less than Pepsico's bid)
- lower values to others than to Coke and Pepsi is consistent with FTC's view that Coke and Pepsi would have gained market power through these mergers

Eventually

- Dr Pepper and Seven-Up merged
  - by 1995: Dr Pepper/Seven-Up: 11.5% of carbonated beverages market
  - Cadbury: 5.5% [Schweppes, Canada Dry, Crush, Sunkist, and A&W (root beer) brands]
  - Cadbury bought Dr Pepper/Seven-Up (17% of soft-drink market, and half non-cola part)
  - Coke: 41%, Pepsi: 32%
- mergers increased share of top 3 firms
- FTC's actions limited share of top 2 firms

The Order of play

- So far it has been assumed that firms choose their output levels **simultaneously.**
- The competition between the firms is then a **simultaneous play game** in which the output levels are the strategic variables.
- What if firm 1 chooses its output level first and then firm 2 responds to this choice?
- Firm 1 is then a leader. Firm 2 is a follower.
- The competition is a **sequential game** in which the output levels are the strategic variables.

Stackelberg games

- Cournot model: both firms make their output decisions simultaneously
- Heinrich von Stackelberg's model: firms act sequentially
  - leader firm sets its output first
  - then its rival (follower) sets its output
- Is it better to be the leader?
- Or is it better to be the follower?

Stackelberg games

- **Q:** What is the best response that follower firm 2 can make to the choice y_1 already made by the leader, firm 1?
- **A:** Choose $y_2 = R_2(y_1)$.
- Firm 1 knows this and so perfectly **anticipates** firm 2's reaction to any y_1 chosen by firm 1.

Stackelberg games

- This makes the leader's profit function

$$\Pi_1^s(y_1) = p(y_1 + R_2(y_1))y_1 - c_1(y_1).$$

The leader chooses y_1 to maximize its profit.

- **Q:** Will the leader make a profit at least as large as its Cournot-Nash equilibrium profit?
- **A:** Yes. The leader could choose its Cournot-Nash output level, knowing that the follower would then also choose its C-N output level. The leader's profit would then be its C-N profit. But the leader does not have to do this, so its profit must **be at least as large as** its C-N profit.

Stackelberg games; An example

- The market inverse demand function is $p = 60 - y_T$. The firms' cost functions are $c_1(y_1) = y_1^2$ and $c_2(y_2) = 15y_2 + y_2^2$.
- Firm 2 is the follower. Its reaction function is

$$y_2 = R_2(y_1) = \frac{45 - y_1}{4}.$$

- The leader's profit function is therefore

$$\Pi_1^s(y_1) = (60 - y_1 - R_2(y_1))y_1 - y_1^2$$

$$= (60 - y_1 - \frac{45 - y_1}{4})y_1 - y_1^2$$

$$= \frac{195}{4}y_1 - \frac{7}{4}y_1^2.$$

Stackelberg games; An example

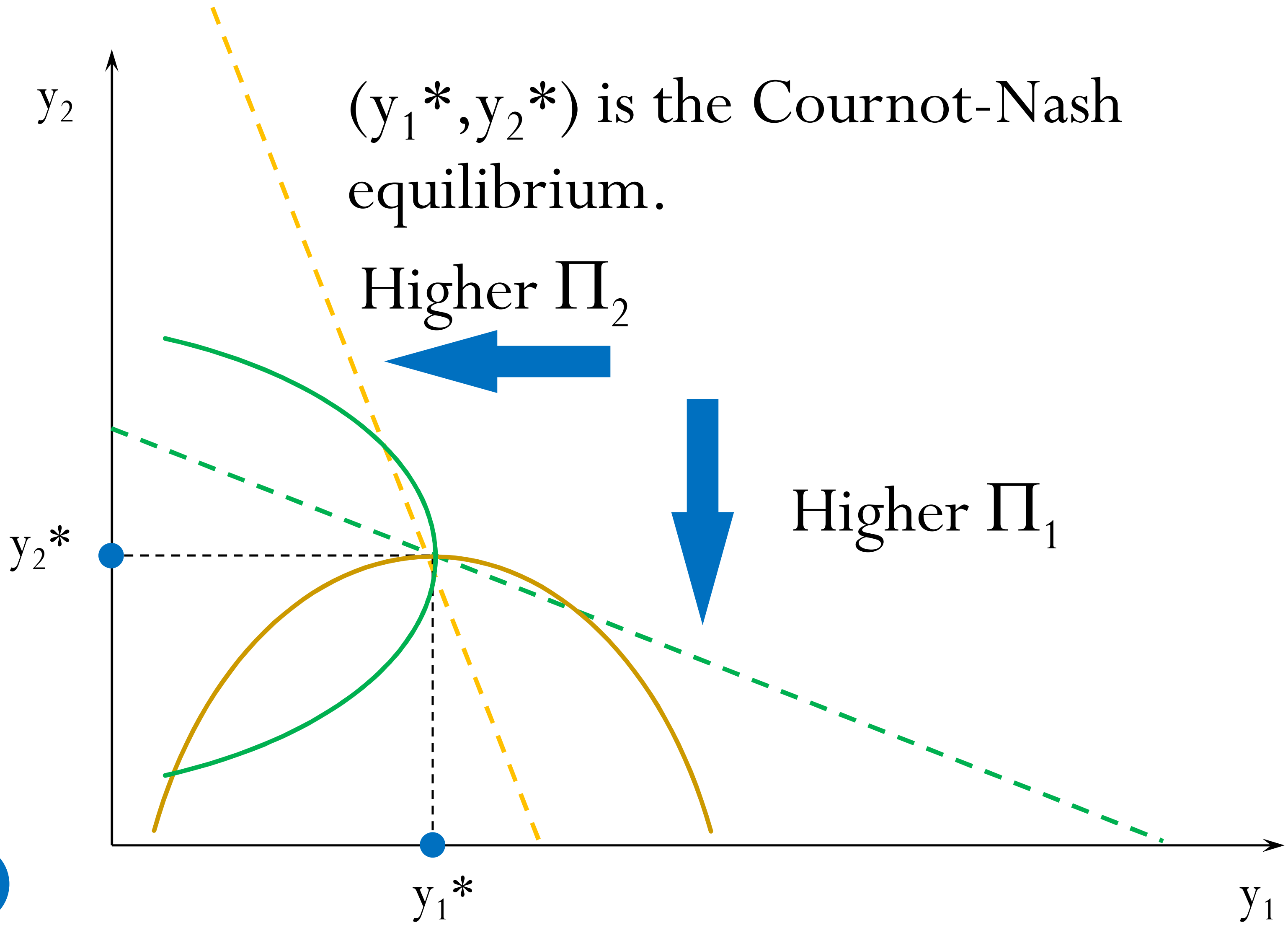
- For a profit-maximum

$$\frac{195}{4} = \frac{7}{2} y_1 \Rightarrow y_1^s = 13 \cdot 9$$

- **Q:** What is firm 2's response to the leader's choice $y_1^s = 13 \cdot 9$?
- **A:** $y_2^s = R_2(y_1^s) = \dfrac{45 - 13 \cdot 9}{4} = 7 \cdot 8.$

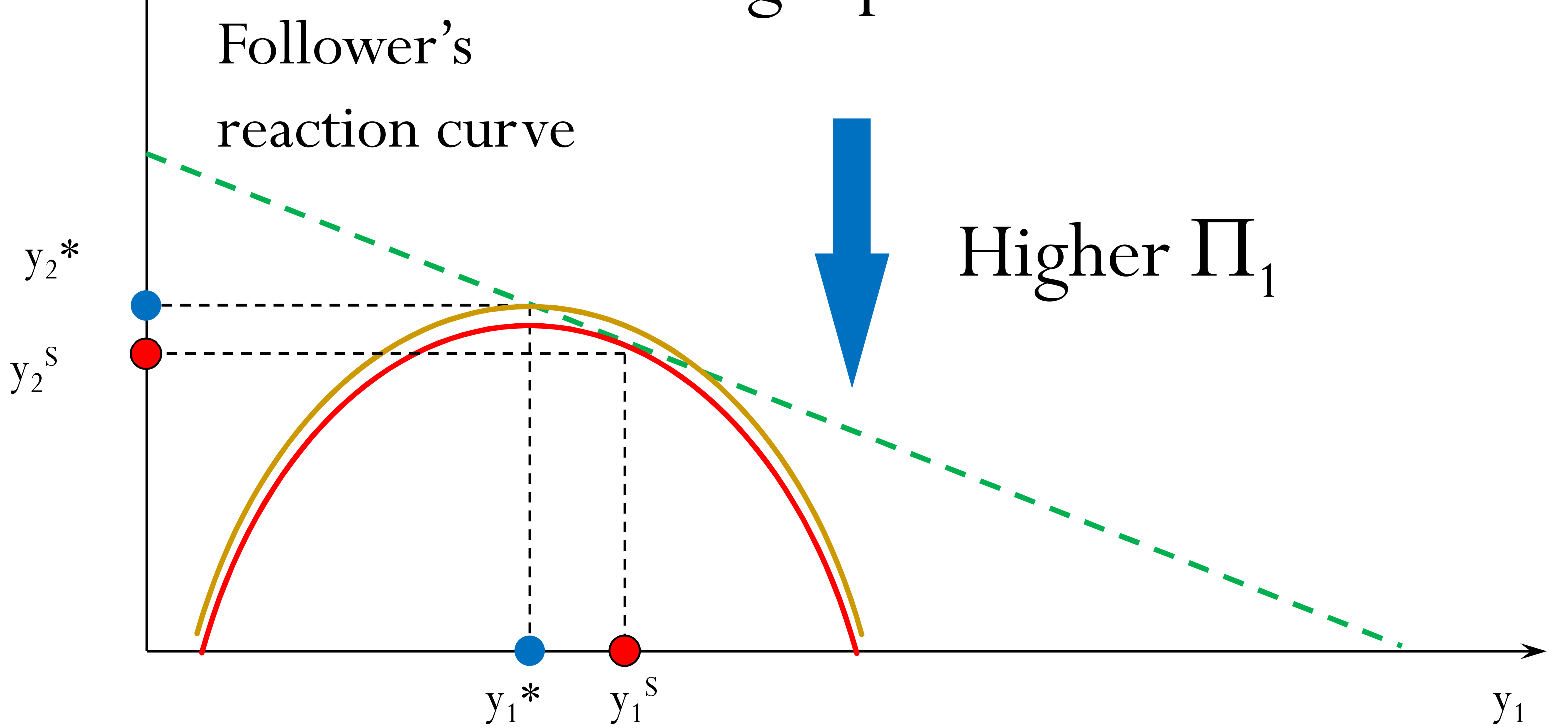
- The C-N output levels are $(y_1^*, y_2^*) = (13,8)$ so the leader produces more than its C-N output and the follower produces less than its C-N output. This is true generally.

Stackelberg games; An example



Stackelberg games; An example

(y_1^*, y_2^*) is the Cournot-Nash equilibrium. (y_1^S, y_2^S) is the Stackelberg equilibrium.



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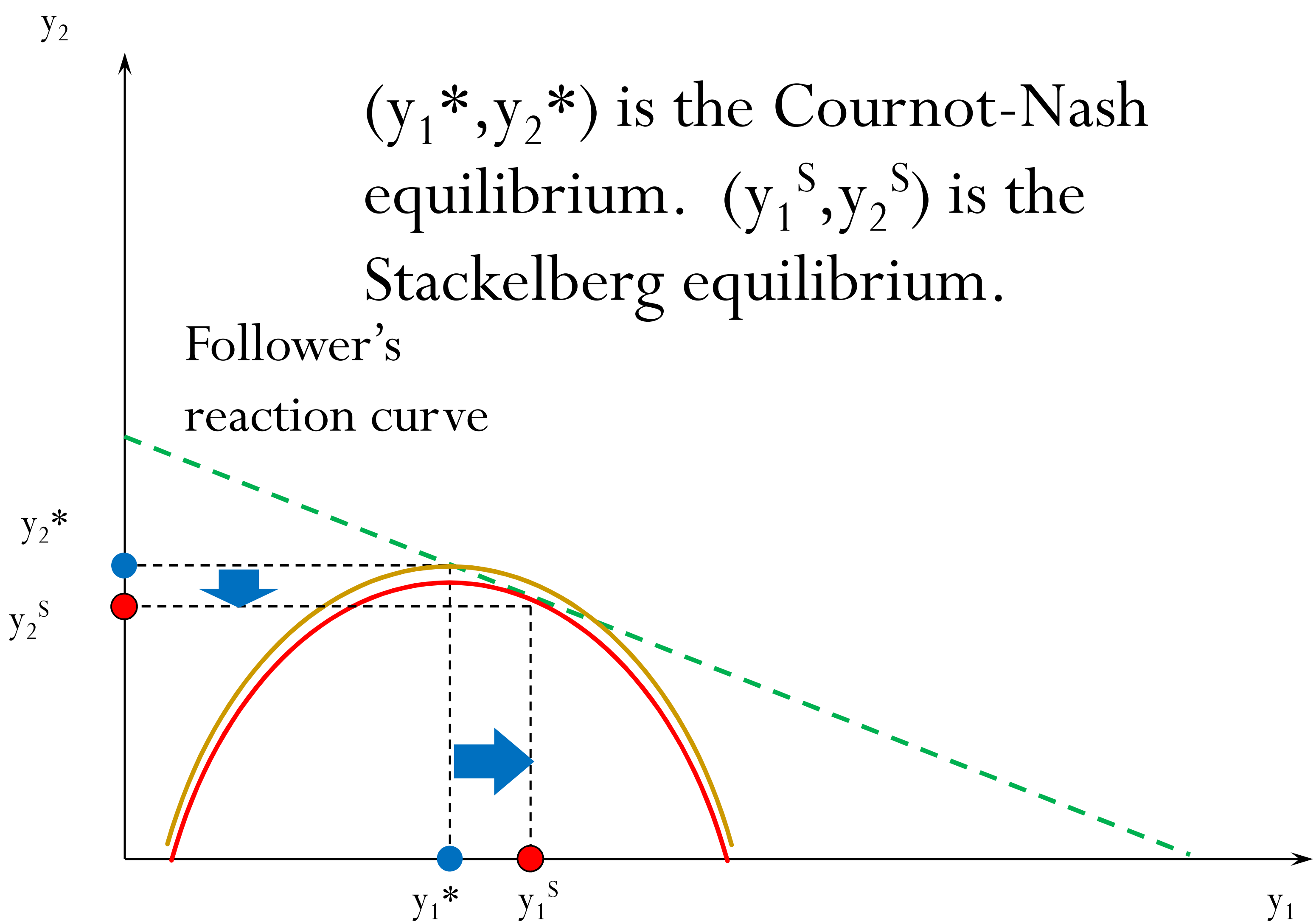

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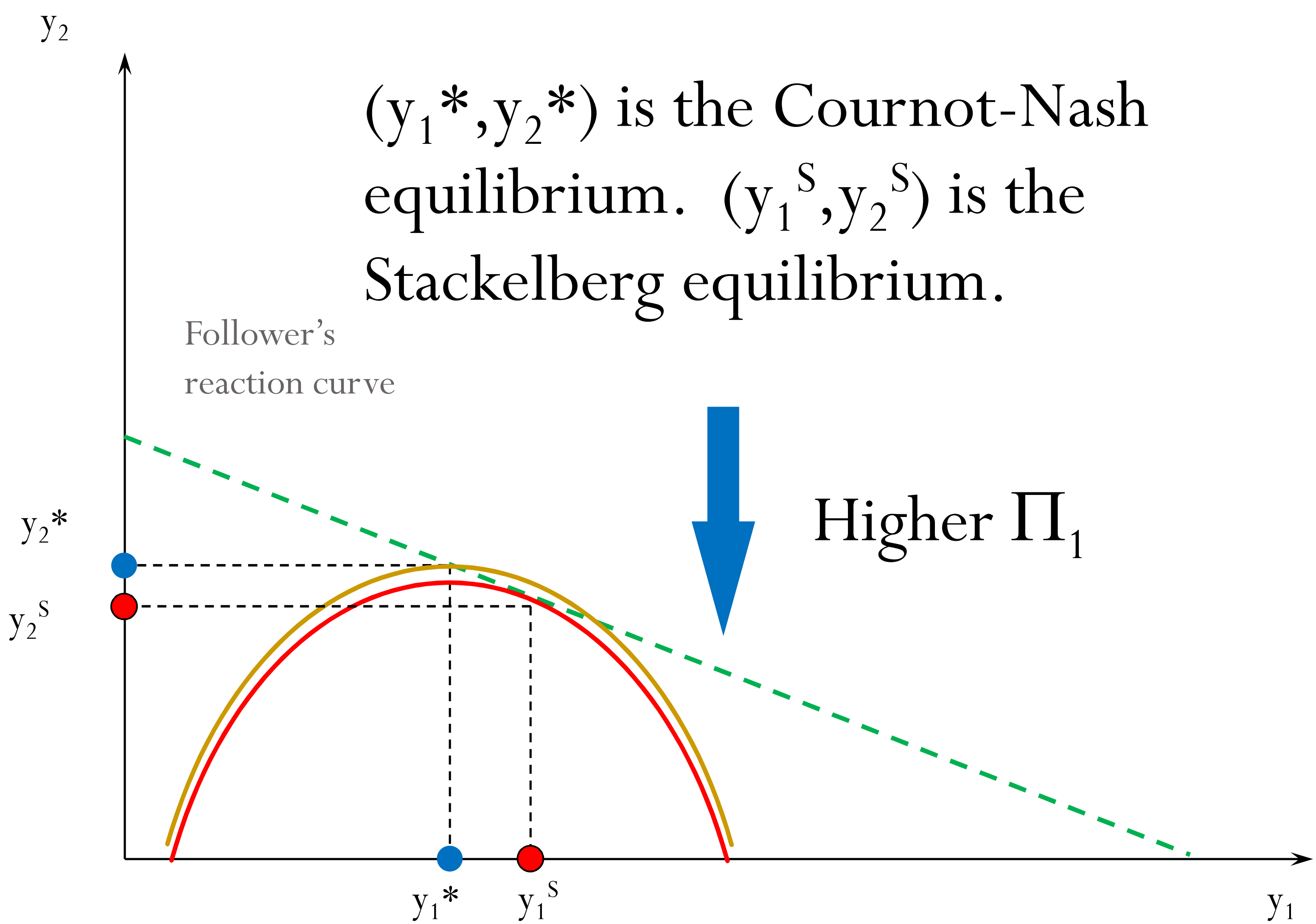

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Game Theory

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Supply Functions Equilibria

- In these models, it is assumed that the amount of energy that a firm is willing to deliver is related to the market price through a supply function:

$q_i = q_i(p)$ for i=1 to n

- In this case, the decision variables of each firm are thus neither the price nor the quantity but the parameters of its supply function.
- At equilibrium, the total demand is equal to the sum of the quantities produced by all the firms:

$$D(p) = \sum_i q_i(p)$$

Supply Functions Equilibria

- The profit of each firm can be expressed as follows:

$$\Pi_i = p \times q_i(p) - C_i(q_i(p))$$

$$q_i(p) = D(p) - \sum_{-i} q_{-i}(p)$$

- These profit functions can be differentiated with respect to the price to get the necessary conditions for optimality, which after some manipulations can be expressed in the following form:

$$q_i(p) = \left(p - \frac{dC_i(q_i(p))}{q_i(p)}\right)\left(\frac{-dD}{dp} + \frac{\sum_{-i} q_{-i}(p)}{dp}\right)$$

$$q_i(p) = \left(p - \frac{dC_i(q_i(p))}{q_i(p)}\right)\left(\frac{-dq_i(p)}{dp}\right)$$

Supply Functions Equilibria

- The solution of this system of equation is an equilibrium point at which all firms simultaneously maximize their profits. These optimality conditions are differential equations because the parameters of the supply functions are unknown. In order to find a unique solution to this set of differential equations, the supply and cost functions are usually assumed to have respectively linear and quadratic forms:

$$q_i(p) = \beta_i(p - \alpha_i)$$

$$Ci(qi)=1/2\ a_i\ q_i^2 + b_i\ (q_i)$$