

# کنترل پیش بین

## Model Predictive Control

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مهندسی برق و کامپیوتر دانشگاه خواجه نصیر

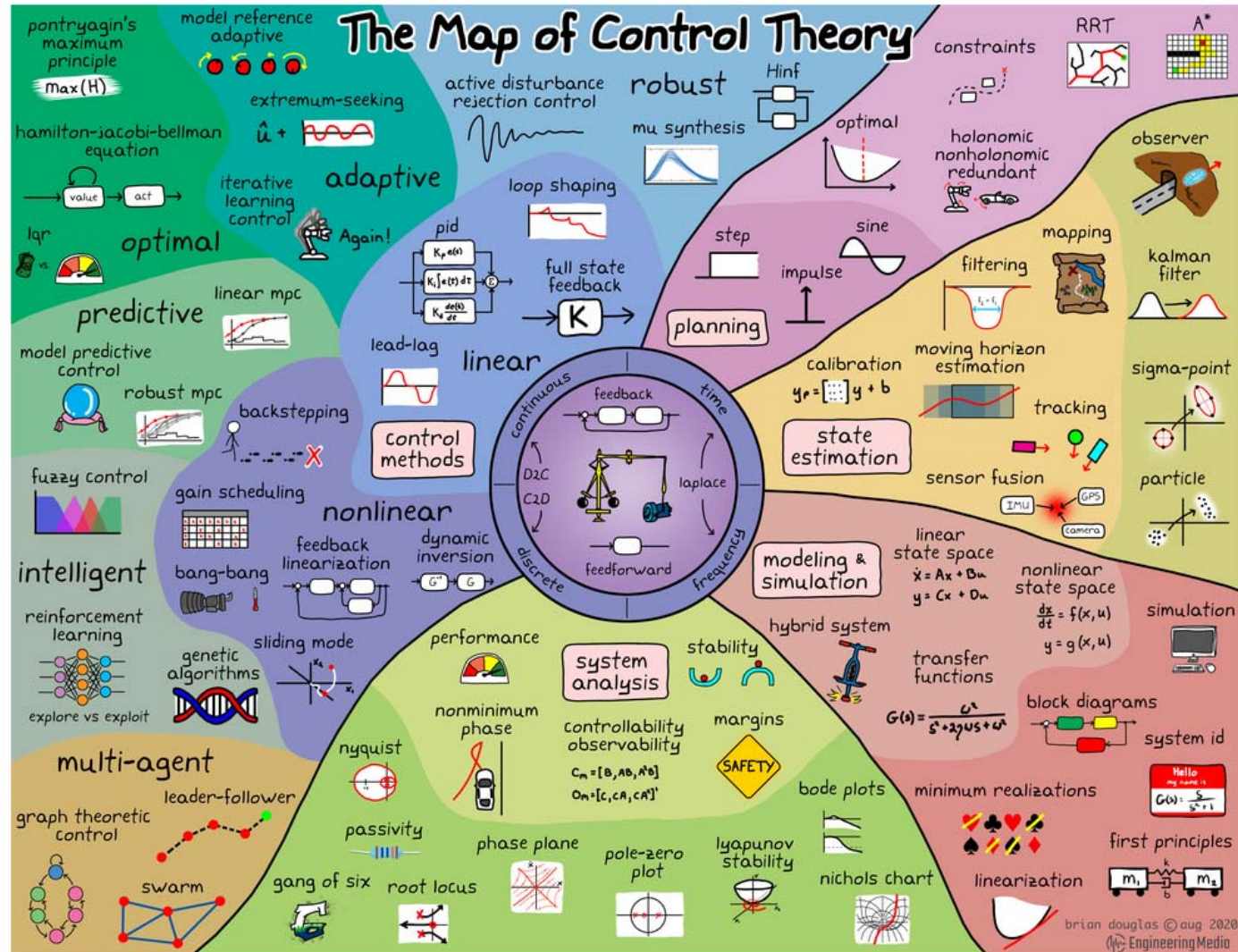


دانشگاه صنعتی خواجه نصیرالدین طوسی

# About the course



- Control



# About the course



Syllabus :

Introduction

Introduction to MPC, typical industrial structure, MPC algorithms architecture

History of MPC

Linear Model Predictive Control design

Steady-state optimization, dynamic optimization

Quick overview of numerical optimization problems

MPC for linear time-invariant discrete-time systems. Implementation in code.

Overview of quadratic programming and the active set method.

# About the course



## □ Linear Model Predictive Control analysis

Asymptotic (exponential) stability analysis

Feasibility and Stability

Stability and Invariance of MPC

Practical Issues

## □ Nonlinear systems

Linear parameter-varying, time-varying, and nonlinear MPC

Moving horizon estimators

## □ Advanced Topics on MPC

Explicit MPC, Economic MPC, Hybrid MPC, Robust MPC,  
Distributed MPC, Stochastic MPC, data-driven MPC

# About the course



- Lectures:
  - Monday, and Wednesday 15:30-17:00
- Office hours:
  - Wednesday 11-13
- Grading
  - Homework 30%
  - Final exam 20%
  - Project (10% bonus) 50%-60%

# Reference

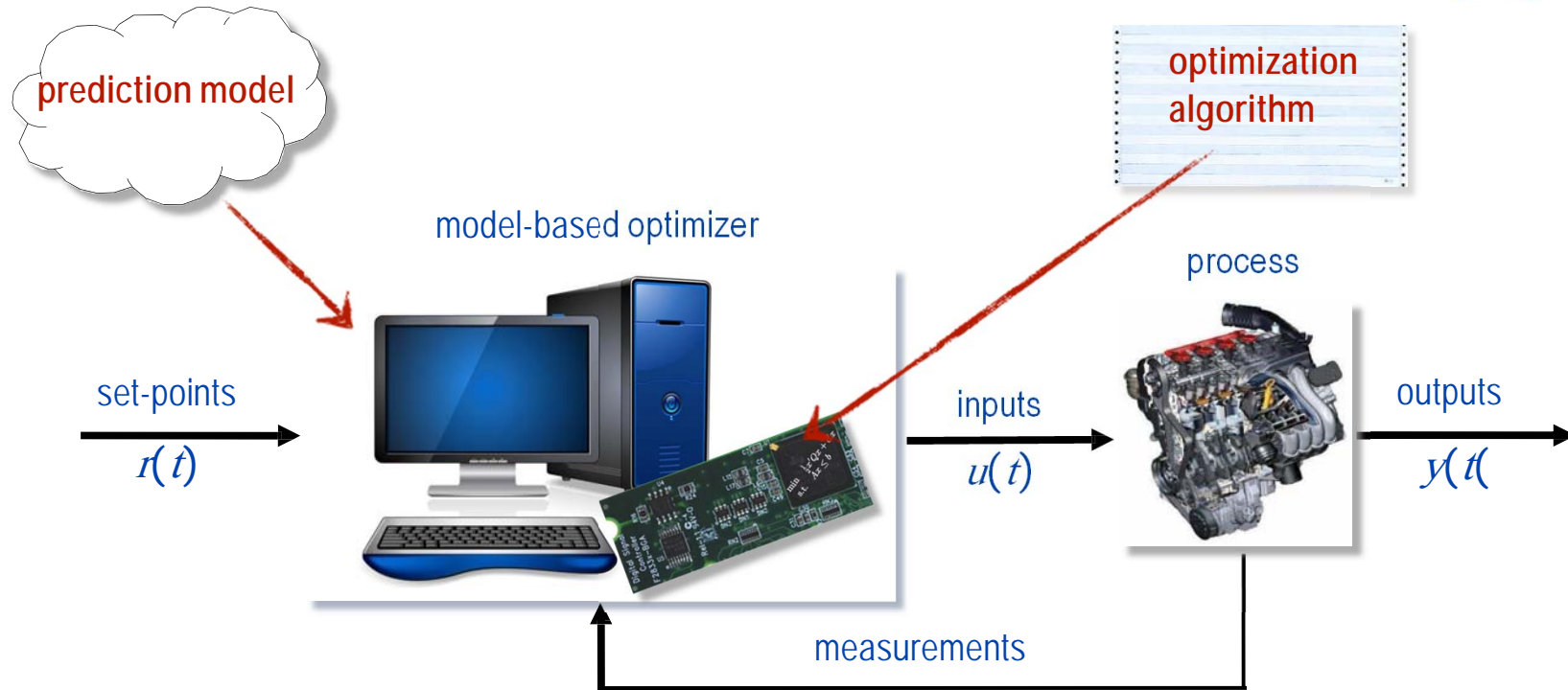


There are many interesting books on MPC.

We will be using the following:

- ❑ Borrelli, F., Bemporad, A. and Morari, M., Predictive control for linear and hybrid systems. Cambridge University Press, 2017.
- ❑ Camacho, E. F., & Alba, C. B. (2013). Model predictive control. Springer
- ❑ Wang, L., 2009. Model predictive control system design and implementation using MATLAB®. Springer .
- ❑ Huang, S., & Lee, T. H. (2013). Applied predictive control. Springer
- ❑ Grüne, L., & Pannek, J. (2017). Nonlinear model predictive control. Springer.

# Model Predictive Control (MPC)



Use a dynamical **model** of the process to **predict** its future evolution and choose the best **control** action

# Model Predictive Control (MPC)

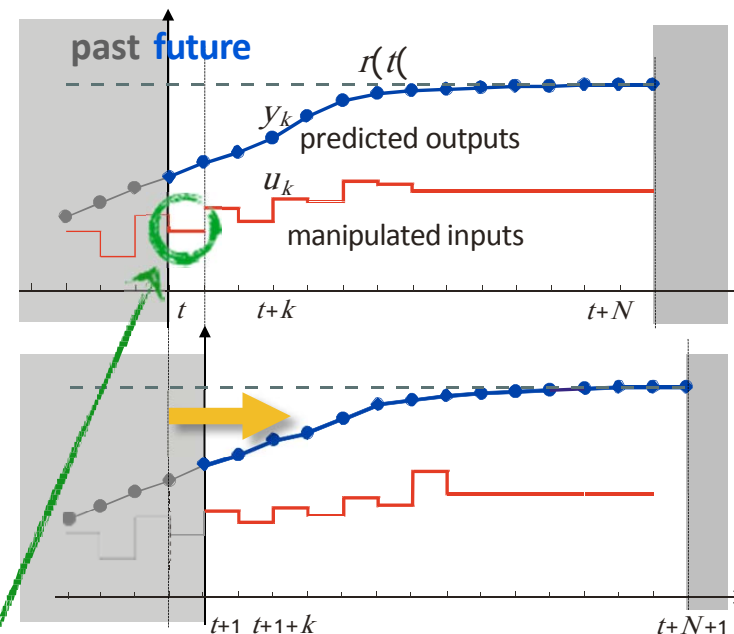


- **Goal:** find the best control sequence over a future horizon of  $N$  steps

$$\min \sum_{k=0}^{N-1} \|W^y(y_k - r(t))\|_2^2 + \|W^u(u_k - u_r(t))\|_2^2$$

s.t.  $x_{k+1} = f(x_k, u_k)$  prediction model  
 $y_k = g(x_k)$   
 $u_{\min} \leq u_k \leq u_{\max}$  constraints  
 $y_{\min} \leq y_k \leq y_{\max}$   
 $x_0 = x(t)$  state feedback

➔ numerical optimization problem



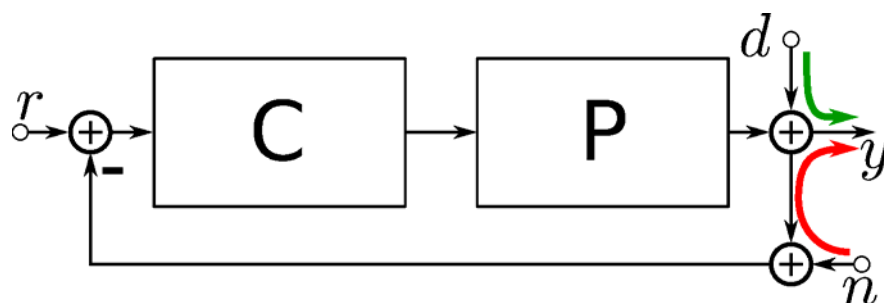
- **At each time  $t$ :**
  - get new measurements to update the estimate of the current state  $x(t)$
  - solve the optimization problem with respect to  $\{u_0, \dots, u_{N-1}\}$
  - apply only the first optimal move  $u(t) = u_0^*$ , discard the remaining samples



# Two Different Perspectives



Classical design: design C

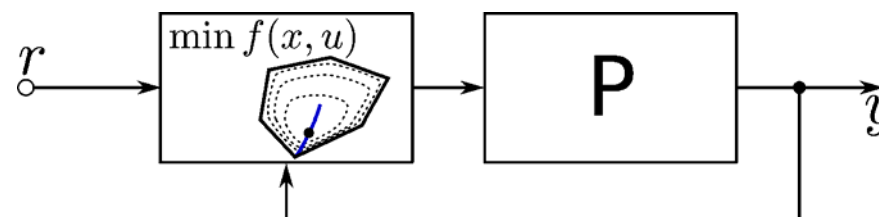


Dominant issues addressed

- Disturbance rejection ( $d$  in  $y$ )
- Noise insensitivity ( $n \rightarrow y$ )
- Model uncertainty

)usually in frequency domain(

MPC: real-time, repeated optimization to choose  $u(l)$



Dominant issues addressed

- Control constraints (limits(
  - Process constraints (safety(
- ) usually in time domain(

# Constraints in Control

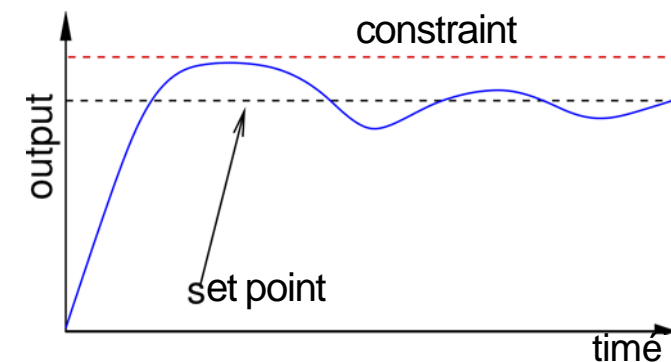
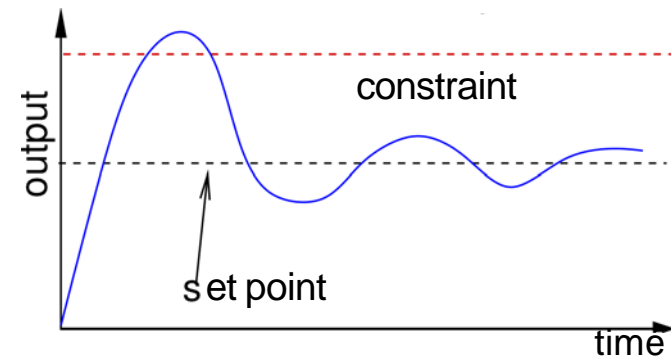


All physical systems have constraints:

- Physical constraints, e.g. actuator limits
- Performance constraints, e.g. overshoot
- Safety constraints, e.g. temperature/pressure limits

Optimal operating points are often near constraints. Classical control methods:

- Adhoc constraint management
- Set point sufficiently far from constraints
- Suboptimal plant operation



Predictive control:

- Constraints included in the design
- Set point optimal
- Optimal plant operation

# Important Aspects of MPC



## □ Main advantages:

- Systematic approach for handling *constraints*
- High *performance* controller

## □ Main challenges:

### □ **Implementation**

MPC problem has to be solved in real-time, i.e. within the sampling interval of the system, and with available hardware (storage, processor,...).

### □ **Stability**

Closed-loop stability, i.e. convergence, is not automatically guaranteed

### □ **Robustness**

The closed-loop system is not necessarily robust against uncertainties or disturbances

### □ **Feasibility**

Optimization problem may become infeasible at some future time step, i.e. there may not exist a plan satisfying all constraints

# Daily-life examples of MPC



- MPC is like playing chess !



- On-line (event-based) re-planning used in GPS navigation



- You use MPC too when you drive !

# Autonomous dNaNo Race Cars



Race car:

- 1:43 scale, very light (50g) and fast
- Radio controlled
- 2.4GHz transmitter allows to run up to 40 cars



Control Problem:

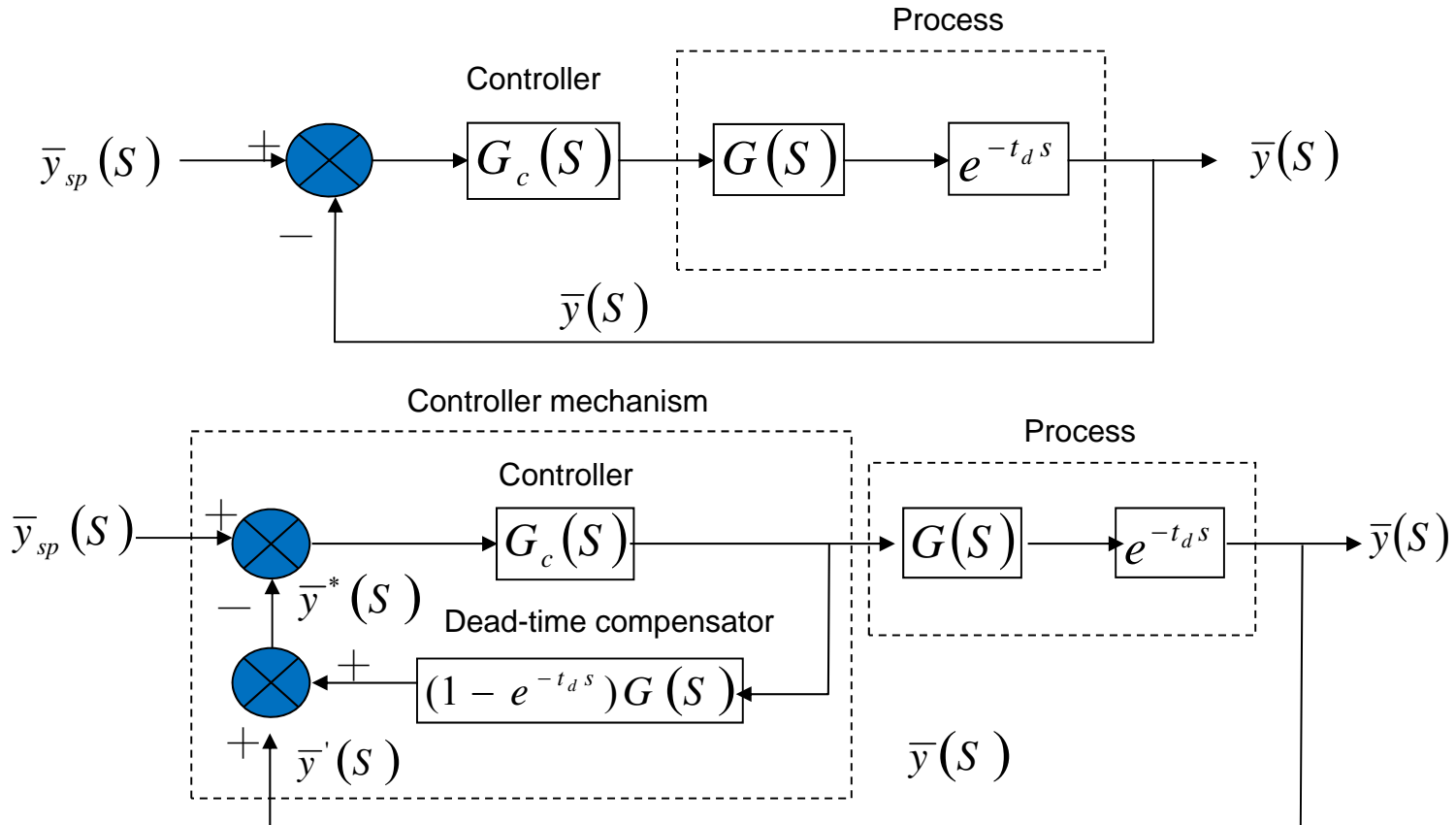
- *Nonlinear model* in 4D (position, orientation)
- *Constraints*: acceleration, steering angle, race track, other cars...
- *Task*: Optimal path planning and path following
- *Challenges*: State estimation, effects that are difficult to model/measure, e.g. slip, small sampling times



# History of MPC



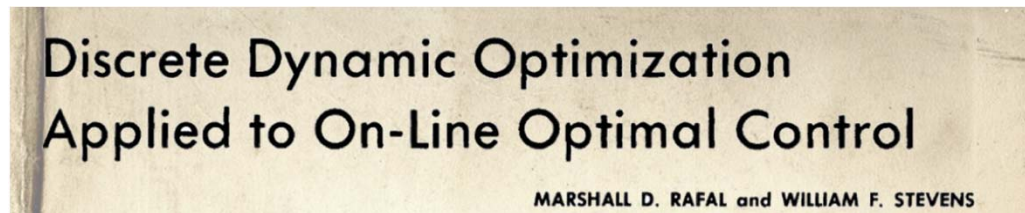
The **Smith Predictor** (perhaps the earliest predictive controller, 1959 by O.J. Smith)



# History of MPC



The MPC concept dates back to the 60's



(Rafal, Stevens, AiChE Journal, 1968)

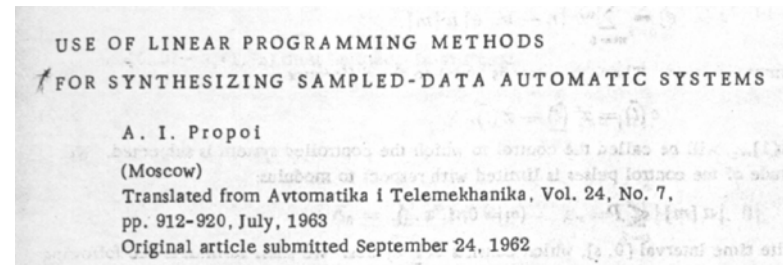


Том XXIV «АВТОМАТИКА И ТЕЛЕМЕХАНИКА» № 7  
1963

УДК 62-50

ПРИМЕНЕНИЕ МЕТОДОВ ЛИНЕЙНОГО ПРОГРАММИРОВАНИЯ  
ДЛЯ СИНТЕЗА ИМПУЛЬСНЫХ АВТОМАТИЧЕСКИХ  
СИСТЕМ

А. И. ПРОПОЙ



(Propoi, 1963)

- MPC used in the process industries since the 80's

(Qin, Badgwell, 2003) (Bauer, Craig, 2008)

Today APC (advanced process control) = MPC



# History of MPC



- ❑ 1970s: Cutler suggested MPC in his Ph D proposal at the University of Houston in 1969 and introduced it later at Shell under the name Dynamic Matrix Control. C. R. Cutler, B. L. Ramaker, 1979 “Dynamic matrix control — a computer control algorithm”. *AIChE National Meeting*, Houston, TX.
  - ❑ successful in the petro-chemical industry
  - ❑ linear step response model for the plant
  - ❑ quadratic performance objective over a finite prediction horizon
  - ❑ future plant output behavior specified by trying to follow the set-point as closely as possible
  - ❑ input and output constraints included in the formulation
  - ❑ optimal inputs computed as the solution to a least—squares problem
  - ❑ adhoc input and output constraints. Additional equation added online to account (or constraints. Hence a dynamic matrix in the least squares problem.
- ❑ C. Cutler, A. Morshedi, J. Haydel, 1983. “An industrial perspective on advanced control”. *AIChE Annual Meeting*, Washington, DC.
  - ❑ Standard QP problem formulated in order to systematically account for constraints.



# History of MPC



- ❑ Mid 1990s: extensive theoretical effort devoted to provide conditions for guaranteeing feasibility and closed-loop stability
- ❑ 2000s: development of tractable robust MPC approaches; nonlinear and hybrid MPC; MPC for very fast systems
- ❑ 2010s: stochastic MPC; distributed large-scale MPC; economic MPC; data driven MPC

# MPC in industry



)Qin, Badgewell, (2003

- Industrial survey of MPC applications conducted in mid 1999

Area	Aspen Technology	Honeywell Hi-Spec	Adersa <sup>b</sup>	Invensys	SGS <sup>c</sup>	Total
Refining	1200	480	280	25		1985
Petrochemicals	450	80	—	20		550
Chemicals	100	20	3	21		144
Pulp and paper	18	50	—	—		68
Air & Gas	—	10	—	—		10
Utility	—	10	—	4		14
Mining/Metallurgy	8	6	7	16		37
Food Processing	—	—	41	10		51
Polymer	17	—	—	—		17
Furnaces	—	—	42	3		45
Aerospace/Defense	—	—	13	—		13
Automotive	—	—	7	—		7
Unclassified	40	40	1045	26	450	1601
Total	1833	696	1438	125	450	4542
First App.	DMC:1985 IDCOM-M:1987 OPC:1987	PCT:1984 RMPCT:1991	IDCOM:1973 HIECON:1986	1984	1985	
Largest App.	283 × 603	85 × 225	—	12 × 31	—	

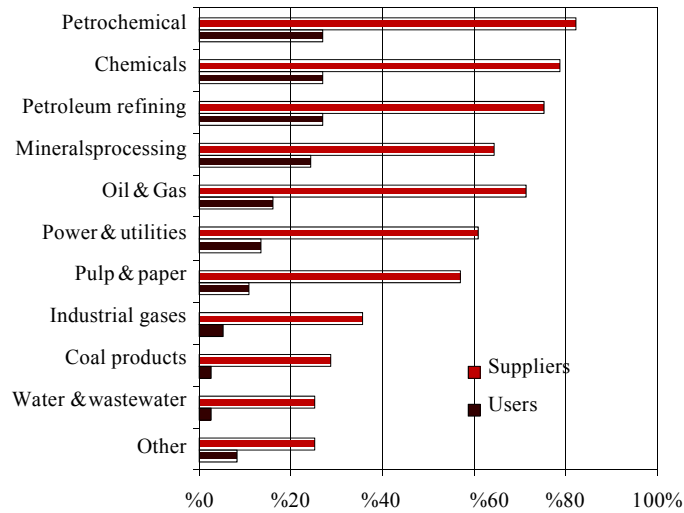
Estimates based on vendor survey

# MPC in industry

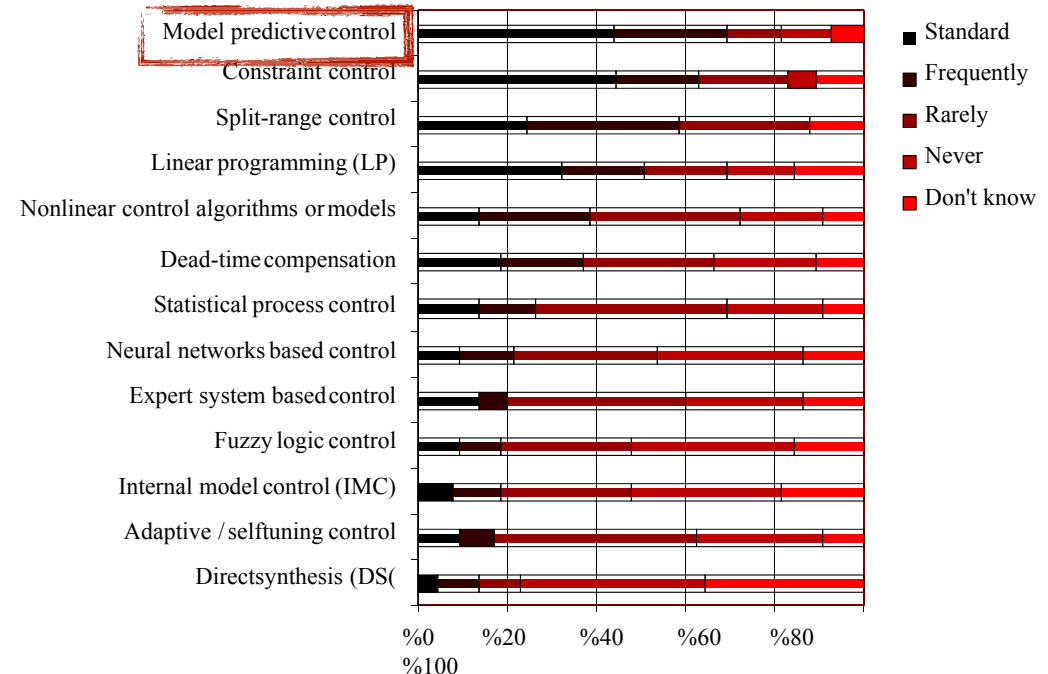


(Bauer, Craig; 2008)

## Economic assessment of Advanced Process Control (APC)



participants of APC survey by industry (worldwide)



Industrial use of APC methods: survey results

# MPC in industry



(Samad, IEEE CS Magazine, 2017)

- Impact of advanced control technologies in industry

**TABLE 1 A list of the survey results in order of industry impact as perceived by the committee members.**

Rank and Technology	High-Impact Ratings	Low- or No-Impact Ratings
PID control	100%	0%
Model predictive control	78%	9%
System identification	61%	9%
Process data analytics	61%	17%
Soft sensing	52%	22%
Fault detection and identification	50%	18%
Decentralized and/or coordinated control	48%	30%
Intelligent control	35%	30%
Discrete-event systems	23%	32%
Nonlinear control	22%	35%
Adaptive control	17%	43%
Robust control	13%	43%
Hybrid dynamical systems	13%	43%

# MPC in industry



)Samad, IFAC Newsletter, April (2019

Control Technology	Current Impact		Future Impact	
	% High	Low/No	High	Low/No
PID control	91%	0%	78%	6%
System Identification	65%	5%	72%	5%
Estimation & filtering	64%	11%	63%	3%
Model-predictive control	62%	11%	85%	2%
Process data analytics	51%	15%	70%	8%
Fault detection & identification	48%	17%	8%	8%
Decentralized and/or coordinated control	29%	33%	54%	11%
Robust control	26%	35%	42%	23%
Intelligent control	24%	38%	59%	11%
Nonlinear control	21%	44%	42%	15%
Discrete-event systems	24%	45%	39%	27%
Adaptive control	18%	38%	44%	17%
Repetitive control	12%	74%	17%	51%
Other advanced control technology	11%	64%	25%	39%
Hybrid dynamical systems	11%	68%	33%	33%
Game theory	5%	76%	17%	52%

# MPC in Aeronautic industry



## PRESS RELEASE

Pratt & Whitney's F135 Advanced Multi-Variable Control Team Receives UTC's Prestigious George Mead Award for Outstanding Engineering Accomplishment

EAST HARTFORD, CONN., THURSDAY, MAY 27, 2010

Pratt & Whitney engineers Louis Celiberti, Timothy Crowley, James Fuller and Cary Powell won the George Mead Award – United Technologies Corp.'s highest award for outstanding engineering achievement – for their pioneering work in developing the world's first advanced multi-variable control (AMVC) design for the only engine that powers the F-35 Lightning II flight test program. Pratt & Whitney is a United Technologies Corp. (NYSE:UTX) company.

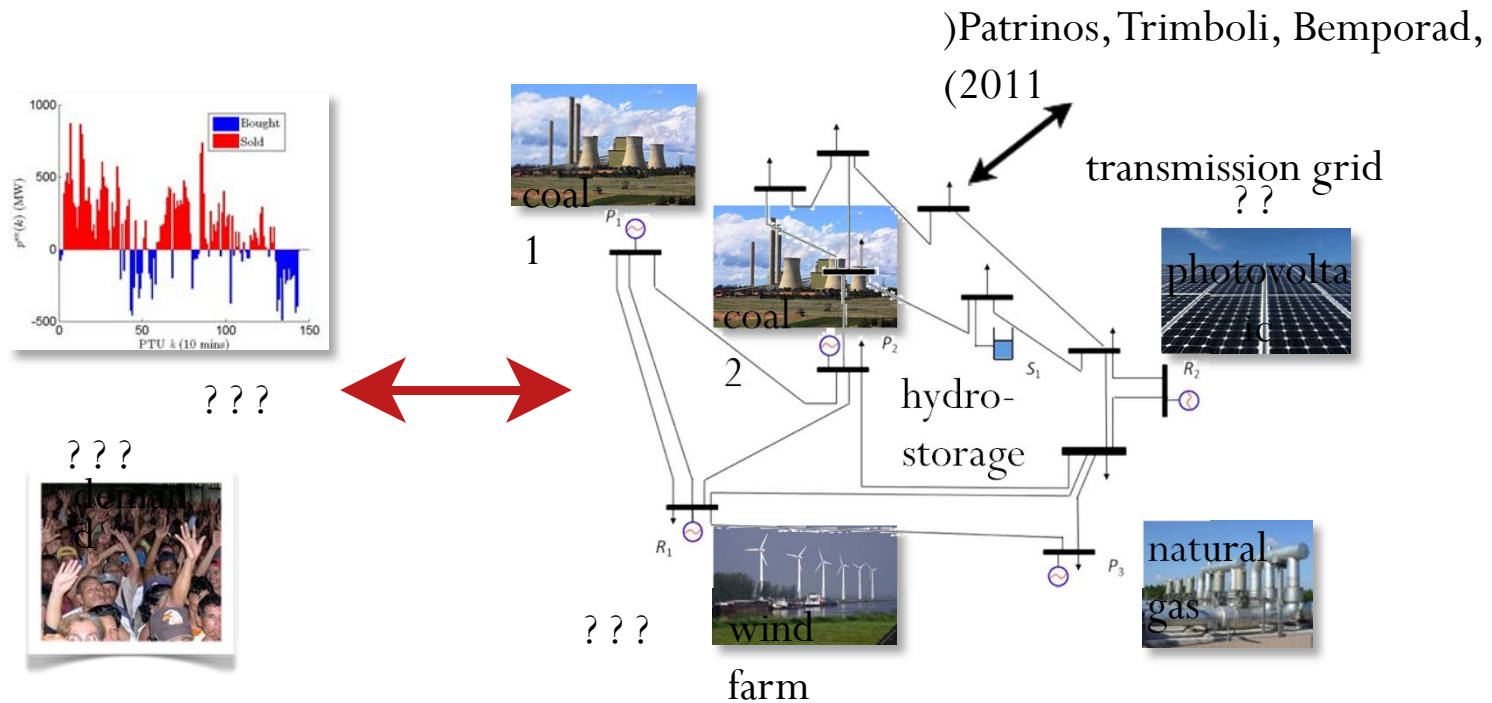
The AMVC, which uses a proprietary model predictive control methodology, is the most technically advanced propulsion system control ever produced by the aerospace industry, demonstrating the highest pilot rating for flight performance and providing independent control of vertical thrust and pitch from five sources. This innovative and industry-leading advanced design is protected with five broad patents for Pratt & Whitney and UTC, and is the new standard for propulsion system control for Pratt & Whitney military and commercial engines.



**Pratt & Whitney**  
A United Technologies Company

<http://www.pw.utc.com/Press/Story/20100527-0100/2010>

# MPC in Smart Electricity Grids



Dispatch power in smart distribution grids, trade energy on energy markets

**Challenges:** account for **dynamics**, network **topology**, physical **constraints**, and **stochasticity** (of renewable energy, demand, electricity prices)

# MPC research is driven by applications



- Process control → **linear** MPC (some **nonlinear** too) 2000-1970
- Automotive control → **explicit, hybrid** MPC 2010-2001
- Aerospace systems and UAVs → **linear time-varying** MPC >2005
- Information and Communication Technologies (ICT)  
(wireless nets, cloud) → **distributed/decentralized** MPC >2005
- Energy, finance, automotive, water → **stochastic** MPC >2010
- Industrial production → **embedded optimization** solvers for MPC 2010<
- Machine learning → **data-driven** MPC today



# Benefits of MPC



- Long history (decades) of success of MPC in industry
- MPC is a **universal control methodology**:
  - to coordinate **multiple inputs/outputs**, arbitrary **models** (linear, nonlinear(... ,
  - to **optimize performance** under **constraints**
  - intuitive to **design**, easy to **calibrate** and **reconfigure** = **short development time**
- MPC is a mature technology also in fast-sampling applications (e.g. **automotive**)
  - modern ECUs can solve MPC problems in **real-time**
  - advanced MPC **software tools** are available for design/calibration/deployment