کنترل پیش بین
Model Predictive Control

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NMPC: dynamics, constraints and cost function

Nonlinear models

- Often **nonlinear models** are available in **continuous time**:

$$\dot{x} = f(x,u)$$
$$y = h(x,u)$$

- For **nonlinear MPC** design, we need a **discrete-time** model:

$$x(k+1) = F(x(k),u(k))$$
$$y(k) = h(x(k),u(k))$$

- Notice that: $F(x(k),u(k)) = x(k) + \int_{t_k}^{t_{k+1}} f(x,u)\,dt$
- For **simplicity**, we use the **notation**: $x^+ = f(x,u)$

Constraints and cost function

- **State and input** constraints: $x(k) \in \mathbf{X}, u(k) \in \mathbf{U}$
- **Stage** cost and **overall cost**: $V_N(x,\mathbf{u}) = \sum_{j=0}^{N-1} l(x(j),u(j)) + V_f(x(N))$

MPC: optimal control problem and assumptions

Main assumptions

- $l(\cdot)$ and $V_f(\cdot)$ are **positive definite**, and $l(0,0)=0$, $V_f(0)=0$
  $f(\cdot)$ is **continuous** and $f(0,0)=0$
- **Control-invariant set** $\mathbf{X}_f \subseteq \mathbf{X}$: For any $x \in \mathbf{X}_f$, there exists $u \in \mathbf{U}$ such that: $V_f(f(x,u)) - V_f(x) \le -l(x,u)$

Optimal control problem

Given the **current state** x, solve:

$$\mathbf{P}_N(x): \quad \min_{\mathbf{u}} V_N(x,\mathbf{u}) \quad \text{s.t.}$$

$$x^+ = f(x,u)$$

$$x(j) \in \mathbf{X} \quad \text{for all } j = 0,\ldots,N-1$$

$$u(j) \in \mathbf{U} \quad \text{for all } j = 0,\ldots,N-1$$

$$x(N) \in \mathbf{X}_f$$

NMPC: a note on computational aspects

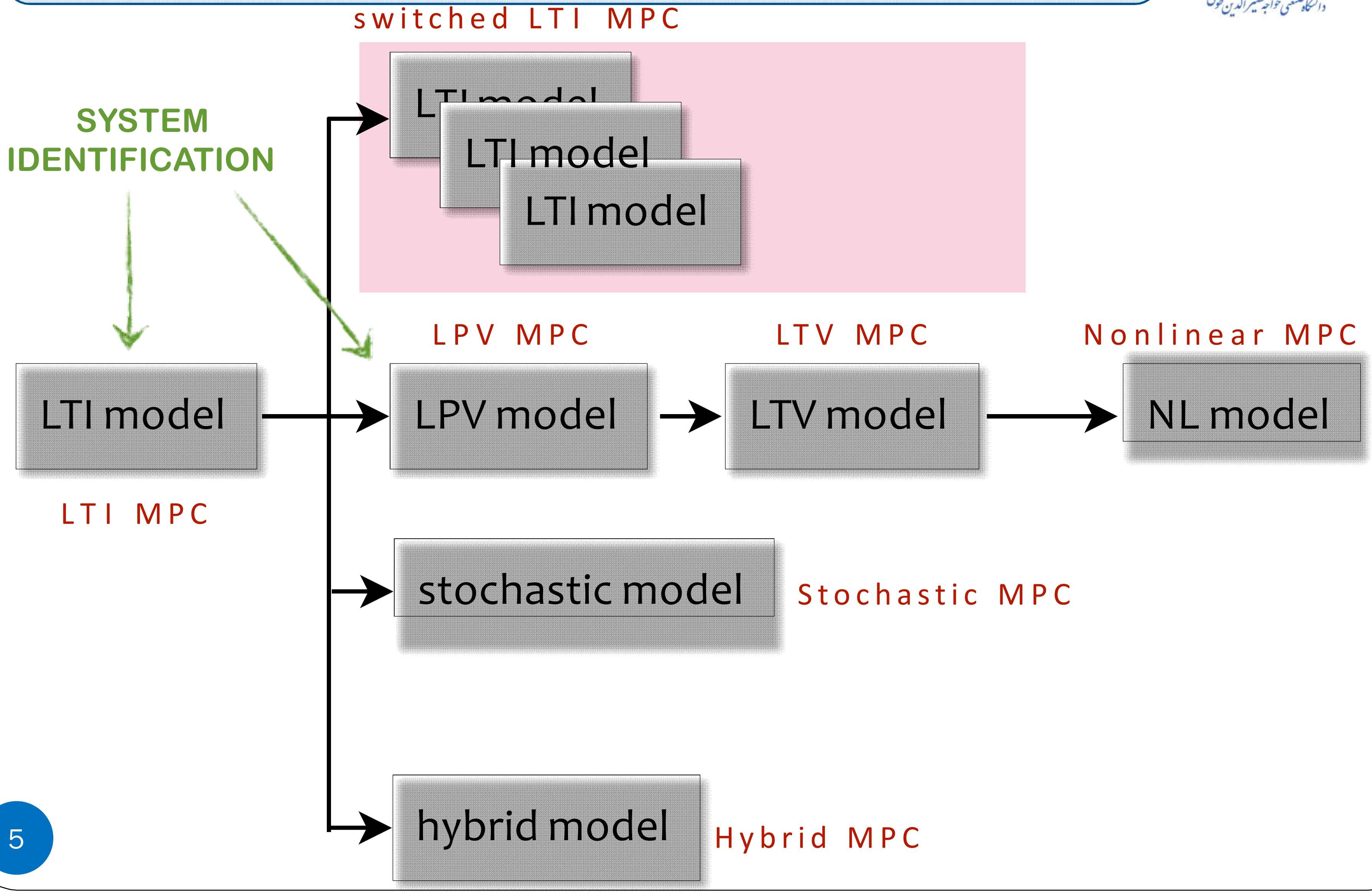
General aspects

- The OCP is a **non-convex**, **nonlinear program**:
  - Computing $f(x, u)$ requires **ODE integration**
  - Finding **global optimum** is difficult
  - Solution algorithms are **time consuming**

Efficient NMPC methods [Diehl et al., 2008]

- Problem formulation aspects:
  - **Sequential**: **eliminate** the **state sequence** and solve for **u**
  - **Simultaneous**: solve for both **state** and **input sequences** (multiple shooting, collocation methods, etc.)
- NLP methods:
  - **Sequential Quadratic Programming**: repeated **linearization** of constraints and **quadratic** expansion of the cost function
  - **Interior Point Methods**: direct solution of the (slightly modified) nonlinear optimality KKT conditions

Summery: Choice of prediction model



Conclusions

- MPC is a **universal control methodology**:
  - different **models** (linear, nonlinear, hybrid, stochastic, …)
  - **optimize** closed-loop performance subject to **constraints**
  - **widely applicable** to many industrial sectors
- **MPC research**:
  1. Linear, uncertain, explicit, hybrid, nonlinear MPC: **mature theory**
  2. Stochastic MPC, economic MPC: **still open issues**
  3. Embedded optimization methods for MPC: **still room for many new ideas**
  4. System identification for MPC: there is **a lot to "learn"** from machine learning
  5. Data-driven MPC: **a lot of open issues**.
- **MPC technology**: mature enough for widespread use in industrial applications

MPC research areas and researchers

Topic	Linear MPC	Nonlinear MPC
MPC formulations	Mayne, Rawlings, Limon, Lazar, Morari, Pannocchia	Magni, Scattolini, De Nicolao, Grüne, Allgöwer, Findeisen, Guay, Kouvaritakis, Henson
MPC stability and robustness	Lazar, Limon, Rossiter, Chisci, Kouvaritakis, Bemporad, Kerrigan, Mayne, Rawlings, Pannocchia	Teel, Scattolini, Rawlings, Magni, De Nicolao, Lazar, Pannocchia
Fast MPC (online)	Kerrigan, Boyd, Diehl, Bemporad, Zavala, Biegler, Morari, Jones, Jorgensen, Wright, Rawlings, Pannocchia	Diehl, Biegler, Wright, Rawlings, Bock, Findeisen
Fast MPC (explicit control laws)	Bemporad, Morari, Goodwin, Borrelli, Pistikopoulos, Moennigmann, Johansen, Rossiter	Pistikopoulos, Morari, Jones, Raimondo

MPC research areas and researchers

Topic	Linear MPC	Nonlinear MPC
Distributed (hierarchical, decentralized) MPC	Rawlings, Wright, Scattolini, Bemporad, Limon, Johansen, Casavola, Christofides, Camponogara, De Schutter, Ferrari-Trecate, Pannocchia	Rawlings, Wright, Allgöwer, Raimondo, Magni, Scattolini
Performance Monitoring	Qin, Rawlings, Patwardhan, Shah, Huang, Seborg, Lee, Pannocchia	?
Constrained state estimation	Rawlings, Mayne, Lee, Diehl, Bitmead, Goodwin	
Identification and input design	Qin, Huang, Zhu, Ljung, Chiuso, Pannocchia	?
Applications	Process industries, Automotive, Aerospace, Finance, Robotics	Chemical processes, Biomedical, Robotics