نظریه بازیها
Game Theory

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Extensive form

Material

- Dynamic Non-cooperative Game Theory: Second Edition
  - Chapter 2.4
- An Introductory Course in Non-cooperative Game Theory
  - Chapter 7

Zero sum games

- Zero sum games
  - Definitions
  - Pure strategies
  - Dominating strategies
  - Mixed strategies
  - Linear programs to find NE
  - **Extensive form and information structure**

Matrix (strategic form)games

- Two-player **simultaneous** move games (both zero and non-zero sum types) can be written in matrix form (also called strategic form) as shown below.

- The strategies of one player form the rows of the matrix, while the strategies of the other player form the columns. Each entry in the matrix represents a possible outcome based on a corresponding selection of strategies.

This is an example of a two-player matrix game where each player has a choice of two possible strategies.		Column Player (player 2)	
		A	B
Row Player (player 1)	X	(m1,m2)	(m3,m4)
	Y	(m5,m6)	(m7,m8)

Extensive form games

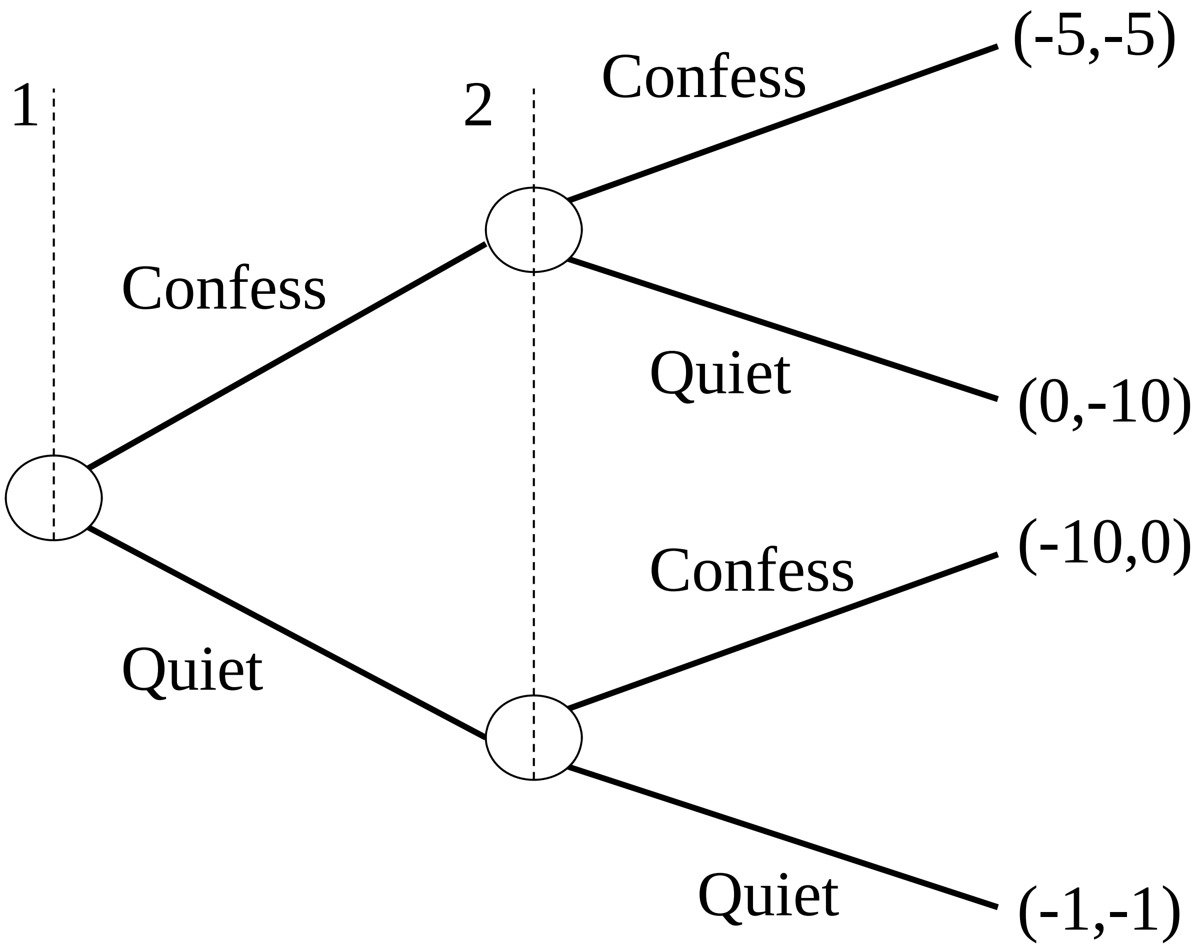
- Who plays when?
- What can they do?
- What do they know?
- What are the payoffs?

Column player's actions

Row player's actions

a_{11}	a_{12}	…	a_{1n}
a_{21}	a_{22}	…	a_{2n}
…			
a_{m1}	a_{m2}	…	a_{mn}

Extensive form games



Extensive form games :example

- **Consider the following setting of the Game:**
- Extensive form?

P_2

P_1

	L	R	
L	1	6	1
M	3	2	(2)
R	0	7	0
	(3)	7	

Extensive form games

- In extensive form, a game is represented with a game tree.
- Extensive form games have the following four elements in common:
  1. Nodes: This is a position in the game where one of the players must make a decision. The first position, called the initial node, is an open dot, all the rest are filled in. Each node is labeled so as to identify who is making the decision.
  2. Branches: These represent the alternative choices that the player faces, and so correspond to available actions.

Extensive form games

3. Payoffs: These represent the pay-offs for each player, with the pay-offs listed in the order of players.
   - When these payoff vectors are common knowledge the game is said to be one of complete information.
   - If, however, players are unsure of the pay-offs other players can receive, then it is an incomplete information game.
4. Information sets: When two or more nodes are joined together by a dashed line this means that the player whose decision it is does not know which node he or she is at. When this occurs the game is characterized as one of imperfect information.
   - When each decision node is its own information set the game is said to be one of perfect information, as all players know the outcome of previous decisions.

Extensive form games

- While the normal form gives the minimum amount of information necessary to describe a game, the extensive form gives additional details about the game concerning the timing of the decisions to be made and the amount of information available to each player when each decision has to be made.
- For every extensive form game, there is one and only one corresponding normal form game. For every normal form game, there are, in general, several corresponding extensive form games.
- Every finite extensive form game of perfect information has a pure strategy Nash equilibrium.

Subgame

- A subgame of a dynamic noncooperative game consists of a single node in the extensive form representation of the game, i.e., the game tree, and all of its successors down to the terminal nodes.
- The information sets and payoffs of a subgame are inherited from the original game.
- Moreover, the strategies of the players are restricted to the history of actions in the subgame.

Subgame Perfect Equilibrium

Backward induction:

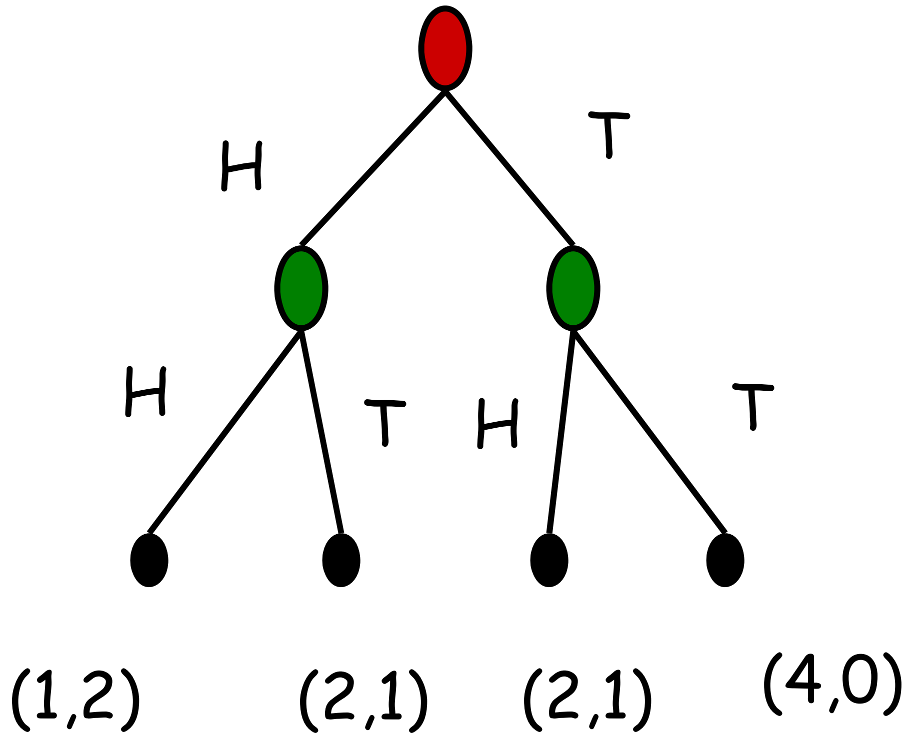
Definition : The **one-stage** deviation principle requires that there must not exist any information set in which a player i can gain by deviating from its subgame-perfect equilibrium strategy (at this **information set**) while its strategy at **other information sets** as well as the **strategies of the other players** are **fixed.**

In other words, a strategy profile s^* is a subgame-perfect equilibrium for each player $i \in N$ and at each information set where player i moves if we:

- fix the other players' strategies as s^*
- fix player i 's moves at other information sets as in s^*

Extensive Form Games

Then, player i cannot improve its payoff (at the information set) by deviating from s_i at the information set only. Note that for games with perfect information the above definitions reduce to **backward induction.**



Finding subgame perfect equilibrium

- Step
  1. Pick a subgame that does not contain any other subgame.
  2. Compute a Nash equilibrium of this subgame.
  3. Assign the payoff vector associated with this equilibrium to the starting node, and eliminate the subgame.
  4. Iterate this procedure until a move is assigned at every contingency, when there remains no subgame to eliminate.
- Nash equilibrium is not necessarily a subgame perfect equilibrium.

backward induction:example

- Consider the following setting of the Game:
- Extensive form?
- Information set?

P_2

P_1

	L	R
L	1	6
M	3	2
R	0	7

1

2

0

3 7

backward induction:example

- Big Monkey and Little Monkey eat warifruit, which dangle from the extreme tip of a lofty branch of the waritree.
- A waritree produces only one fruit. To get the warifruit, at least one of the monkeys must climb the tree and shake the branch bearing the fruit until the fruit comes loose and falls to the ground.
- A warifruit is worth 10 calories of energy. Climbing the tree uses 2 calories for Big Monkey, but uses no energy for Little Monkey, who is smaller. If Little Monkey climbs the tree and shakes it down, Big Monkey will eat 90% of the fruit (or 9 calories) before Little Monkey climbs back down, and Little Monkey will get only 10% of the fruit (or 1 calorie).
- If Big Monkey climbs the tree and Little Monkey waits, Little Monkey will get 40% of the fruit and Big Monkey will get 60%. If both monkeys climb the tree, Big Monkey will get 70% of the fruit and Little Monkey will get 30%. Assume each monkey is simply interested in maximizing his caloric intake.

backward induction:example

- Big Monkey (BM) – Little Monkey (LM)
- Warifruit from waritree (only one per tree) = 10 Calories
- Climb the tree to get the fruit
- Cost to get the fruit :
  - 2 Calories for Big Monkey
  - zero for Little Monkey
- Payoff :
  - Both climb : BM 7 Calories – LM 3 Calories
  - BM climbs : BM 6 Calories – LM 4 Calories
  - LM climbs : BM 9 Calories – LM 1 Calories
- What will they do to maximize payoff taking into account cost?

backward induction:example

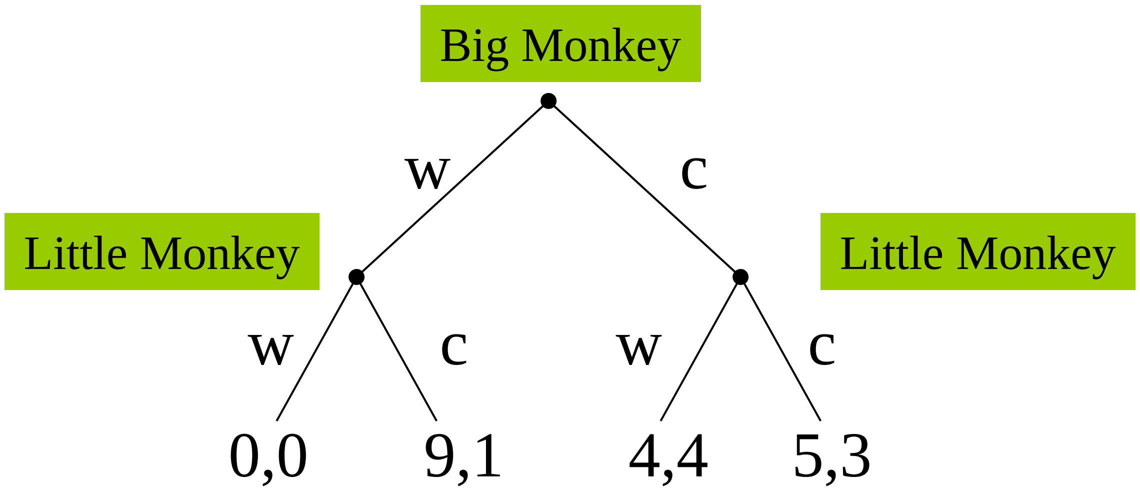
Each monkey can decide to climb the tree or wait at the bottom.

a. What is likely to happen if Big Monkey makes his decision first?

b. What is likely to happen if Little Monkey must decide first?

c. What if they both decide simultaneously?

- BM decides first :



backward induction:example

- Strategies :
  - BM :
    - Wait (w)
    - Climb (c)
  - LM : Actions are ordered, depending on (w,c) of BM
    - Climb no matter what BM does (cc)
    - Wait no matter what BM does (ww)
    - Do the same thing BM does (wc)
    - Do the opposite of what BM does (cw)

- A series of actions that fully define the behavior of a player = strategy.
- A strategy for a player is a complete plan of how to plan the game and prescribes his choices at every information set (in this case, node).

backward induction:example

- Another way to depict the BM-LM game (where BM chooses first) :

  LM : Actions are ordered, depending on (w,c) of BM

  - Climb no matter what BM does (cc)
  - Wait no matter what BM does (ww)
  - Do the same thing BM does (wc)
  - Do the opposite of what BM does (cw)

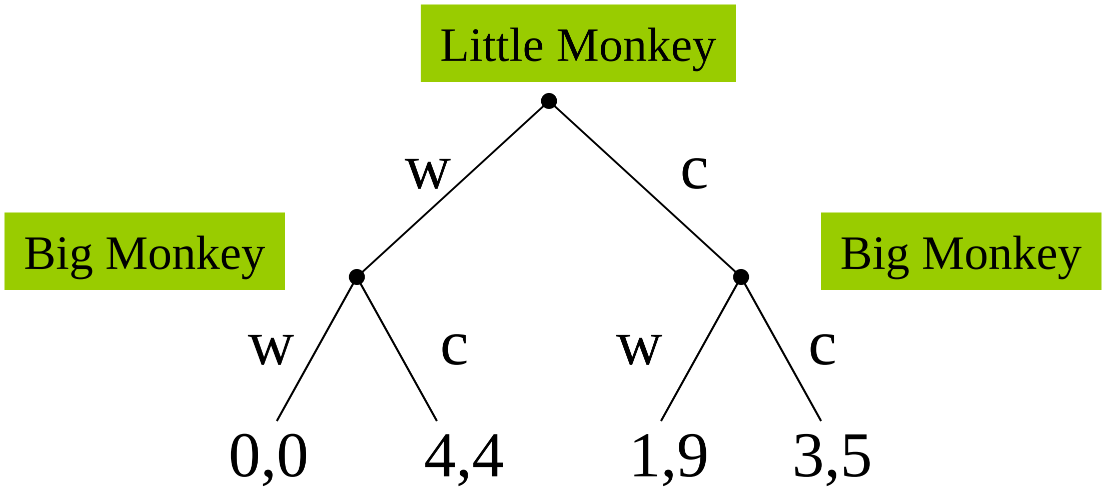
LM

BM

	cc	cw	wc	ww
w	9,1	9,1	0,0	0,0
c	5,3	4,4	5,3	4,4

backward induction:example

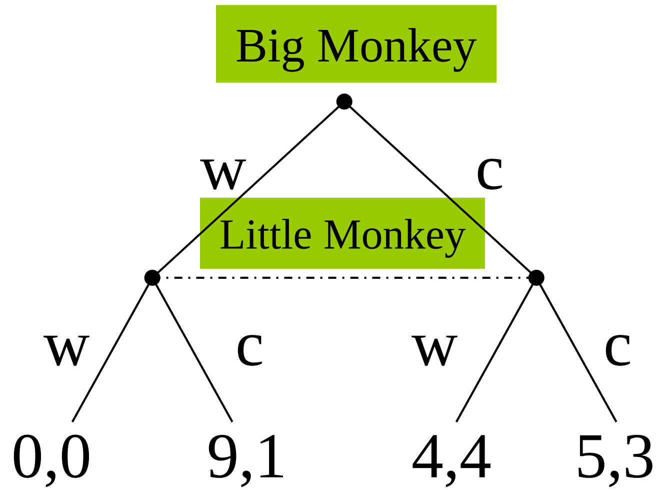
- b)LM decides first :



- The strategies are conversed payoff=(LM, BM)

backward induction:example

- They choose simultaneously :



LM

BM	c	w
c	5,3	4,4
w	9,1	0,0

backward induction:example