دکوپله‌سازی سیستم‌های چندمتغیره با فیدبک حالت

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گروه کنترل- مهر ۱۳۹۸

ELMER GRANT GILBERT (1930-2019)

Elmer Grant Gilbert died 16 June 2019 in Ann Arbor, Michigan, age 89, from congestive heart failure. Elmer was Professor Emeritus of Aerospace Engineering at the University of Michigan. He was born on 29 March 1930 in Joliet, Illinois. He was a member of the National Academy of Engineering and a recipient of the 1994 IEEE Control Systems Award (the citation reads: "for pioneering and innovative contributions to linear state space theory and its applications, especially **realization** and **decoupling**, as well as to control algorithms") and the 1996 Richard E. Bellman Control Heritage Award from the American Automatic Control Council.

• مقدمه

- سیستم‌های دکوپله قطری‌اند.
- با قطری‌سازی سیستم چندمتغیره می‌توان از کنترل‌کننده‌های یک ورودی-یک خروجی استفاده کرد.
- دکوپله سازی سیستم های چندمتغیره با فیدبک حالت

- **کنترل دکوپله سازی**

مساله طراحی: طراحی دو مرحله ای

مرحله اول: دکوپله سازی ماتریس تابع تبدیل سیستم با پیش جبران سازی یا فیدبک و به دست آوردن ماتریس تابع تبدیل قطری.

مرحله دوم: طراحی جبران سازهای مناسب در هر کانال کنترلی.

فرض کنید که:

$$\dot{x}(t) = Ax(t) + Bu(t)$$
$$y(t) = Cx(t) + Du(t)$$
$$A \in R^{n\times n}, B \in R^{n\times m}, C \in R^{m\times n}$$
$$\text{rank}B = \text{rank}C = m$$

با فرض معکوس پذیری ماتریس تابع تبدیل سیستم مربعی داریم:

$$G(s) = C(sI - A)^{-1}B + D$$
$$|D| \neq 0 \Rightarrow G^{-1}(s) = -D^{-1}C(sI - A + BD^{-1}C)^{-1}BD^{-1} + D^{-1}$$

با این فرضیات می توان از معکوس ماتریس تابع تبدیل سیستم مربعی (*ماتریس گویا*) برای پیش جبران سازی استفاده کرد.

اگر

$$\left|D\right| = 0$$

با فرض معکوس پذیری ماتریس تابع تبدیل سیستم مربعی، معکوس آن ناسره است. برای حل این مشکل اندیس های زیر را تعریف کنید:

$$d_i \geq 0(i = 1,\cdots,m), \text{ s.t. } D_d(s) = diag\left\{s^{d_1},\cdots,s^{d_m}\right\}$$

$$\text{is s.t. } B^* = \lim_{|s|\to\infty} D_d(s)G(s)$$

exists and every row of B^* has at least one non zero element.

این اندیس ها یکتا هستند و برابر با کم ترین درجه نسبی توابع تبدیل در هر ردیف از ماتریس تابع تبدیل هستند.

این اندیس ها از تحقق فضای حالت به صورت زیر قابل محاسبه اند:

$$d_i = \begin{cases} 0 & \text{if the ith row of } D \neq 0 \\ \min\left\{k > 0 \middle| c_i^T A^{k-1} B \neq 0\right\} & \text{otherwise} \end{cases}$$

این اندیس ها را اندیس های دکوپله سازی (Decoupling indices) می نامند. و

$$d_i \leq n$$

ماتریس به دست آمده از اندیس های دکوپله سازی زیر را ماتریس دکوپله سازی (Decoupling matrix) می نامند:

$$B^* = \begin{bmatrix} c_1^T A^{d_1 - 1} B \\ c_2^T A^{d_2 - 1} B \\ \vdots \\ c_m^T A^{d_m - 1} B \end{bmatrix}$$

نکته مهم: تعبیر حوزه فرکانسی و زمانی از

$$B^*, G(j\omega_b), G(0)$$

مثال ۴–۲۰، ۴–۲۱

قضیه اگر

$$\left|B^*\right| \neq 0$$

آنگاه کنترل فیدبک حالت زیر

$$u(t) = \left(B^*\right)^{-1}\left[-\mathrm{B}x(t) + v(t)\right]$$

که در آن

$$\mathrm{B} = \begin{bmatrix} c_1^T A^{d_1} \\ c_2^T A^{d_2} \\ \vdots \\ c_m^T A^{d_m} \end{bmatrix}$$

سیستم را دکوپله می سازد و ماتریس تابع تبدیل سیستم دکوپله شده عبارت است از:

$$D_d^{-1}(s) = diag\left\{s^{-d_1}, \cdots, s^{-d_m}\right\}$$

اثبات. داریم

$$\dot{y} = C\dot{x} = CAx + CBu$$

$$\ddot{y} = C\ddot{x} = CA\dot{x} + CB\dot{u} = CA^2x + CABu + CB\dot{u}$$

$$\vdots$$

با توجه به تعریف اندیس های دکوپله سازی، و بررسی هر کدام از خروجی ها در بردار خروجی داریم

$$\begin{bmatrix} \dfrac{d^{d_1} y_1}{dt^{d_1}} \\ \dfrac{d^{d_2} y_2}{dt^{d_2}} \\ \vdots \\ \dfrac{d^{d_m} y_m}{dt^{d_m}} \end{bmatrix} = \begin{bmatrix} c_1^T A^{d_1} \\ c_2^T A^{d_2} \\ \vdots \\ c_m^T A^{d_m} \end{bmatrix} x + \begin{bmatrix} c_1^T A^{d_1 -1} B \\ c_2^T A^{d_2 -1} B \\ \vdots \\ c_m^T A^{d_m -1} B \end{bmatrix} u$$

$$= \overline{\mathrm{B}} x + B^* u$$

اگر ورودی جدید را به صورت زیر تعریف کنیم:

$$v = D_d(s)y \quad \text{or} \quad y = D_d^{-1}(s)v$$

توجه کنید که سیستم با ورودی جدید قطری است و با استفاده از تحقق فضای حالت داریم

$$\dot{x} = Ax + Bu$$

$$v = \mathrm{B}x + B^{*}u$$

که قانون کنترل زیر را می دهد:

$$u = \left(B^{*}\right)^{-1}\left[-\mathrm{B}x + v\right]$$

سیستم داده شده زیر را در نظر بگیرید:

$$\dot{x}(t) = Ax(t) + Bu(t)$$
$$y(t) = Cx(t) + Du(t)$$

با اعمال فیدبک حالت دکوپله ساز داریم

$$\dot{x} = \left(A - B\left(B^*\right)^{-1}\mathrm{B}\right)x + B\left(B^*\right)^{-1}v$$
$$y = \left(C - D\left(B^*\right)^{-1}\mathrm{B}\right)x + D\left(B^*\right)^{-1}v$$

که یک تحقق از رابطه ورودی-خروجی زیر است:

$$y = D_d^{-1}(s)v$$

- **چند نکته:**
  - تمام قطب های سیستم دکوپله شده در مبدا قرار دارند.
  - سیستم دکوپله شده صفر انتقال محدود ندارد.
  - تمام صفرهای انتقال محدود سیستم جبران نشده با قطب های جبران ساز حذف شده اند..
  - صفرهای انتقال نیمه راست صفحه یک محدودیت کلیدی.
  - مرحله دوم طراحی: براورده کردن ملزومات حلقه بسته.

• یک مثال

$$G(s)=\frac{1}{s^2}\begin{bmatrix} s & 1 \\ -1 & 1 \end{bmatrix}$$

$$\Rightarrow M(s)=\begin{bmatrix} \frac{1}{s^2} & 0 \\ 0 & \frac{s+1}{s^2} \end{bmatrix} \Rightarrow \text{ MP Plant}$$

A Minimal Realization:

$$A=\begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, B=\begin{bmatrix} 0 & 1 \\ 1 & 0 \\ -1 & 1 \\ 0 & 0 \end{bmatrix}, C=\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1, \end{bmatrix}, D=\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$c_1^T B = \begin{bmatrix} 1 & 0 \end{bmatrix} \Rightarrow d_1 = 1$$

$$c_2^T B = \begin{bmatrix} 0 & 0 \end{bmatrix}, \Rightarrow c_2^T AB = \begin{bmatrix} -1 & 1 \end{bmatrix} \Rightarrow d_2 = 2$$

$$\Rightarrow B^* = \begin{bmatrix} c_1^T B \\ c_2^T AB \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}, \quad \text{which is nonsingular,}$$

and

$$\mathrm{B} = \begin{bmatrix} c_1^T A \\ c_2^T A^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

and

$$u = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}^{-1} \left(-\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} x + v \right)$$

And the compensated plant is:

$$A = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, B = \begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 0 & -1 \\ 0 & 0 \end{bmatrix}, C = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1, \end{bmatrix}, D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

which gives:

$$D_d^{-1}(s) = \begin{bmatrix} \frac{1}{s} & 0 \\ 0 & \frac{1}{s^2} \end{bmatrix}$$

▪ دکوپله‌سازی با تابع تبدیل حلقه بسته پایدار

- قرار دهید:

$$\widehat{\mathrm{D}}_d(s)=\begin{bmatrix}\mathrm{p}_1(s) & & \circ \\ & \ddots & \\ \circ & & \mathrm{p}_m(s)\end{bmatrix}$$

که در آن $\mathrm{p}_i(s)$ها چند جمله‌ای پایدار دلخواه از درجه‌ی d_i هستند.
می‌توان نشان داد که:

$$\lim_{|s|\to\infty}\widehat{\mathrm{D}}_d(s)\mathrm{G}(s)=\mathrm{B}^*$$

یک تحقق از $\widehat{\mathrm{D}}_d(s)\mathrm{G}(s)$

$$\{\mathrm{A}, \mathrm{B}, \hat{\mathrm{C}}, \mathrm{B}^*\}$$

و فیدبک حالت زیر

$$\mathbf{u}(t) = (\mathrm{B}^*)^{-1}\left[-\hat{\mathrm{C}}\mathbf{x}(t) + \mathbf{v}(t)\right]$$

تابع تبدیل حلقه بسته زیر را می‌دهد:

- **ادامه مثال** – انتخاب کنید:

$$\hat{D}_d(s) = \begin{bmatrix} s+1 & 0 \\ 0 & (s+1)^2 \end{bmatrix}$$

$$\Rightarrow \quad \hat{D}_d(s)G(s) = \frac{1}{s^2}\begin{bmatrix} s(s+1) & s+1 \\ (s+1)^2 & -(s+1)^2 \end{bmatrix} =$$

$$\begin{bmatrix} \dfrac{1}{s} & \dfrac{s+1}{s^2} \\ \dfrac{2s+1}{s^2} & \dfrac{2s+1}{s^2} \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 1 & -1 \end{bmatrix}$$

$$\Rightarrow \quad A = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 1 & -1 \\ 0 & 0 \end{bmatrix} \quad \hat{C} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 2 & 1 \end{bmatrix}$$

$$\mathbf{u}(t) = \begin{bmatrix} 1 & \circ \\ 1 & -1 \end{bmatrix}^{-1} \left(-\begin{bmatrix} 1 & 1 & \circ & \circ \\ \circ & \circ & 2 & 1 \end{bmatrix} \mathbf{x}(t) + \mathbf{v}(t) \right)$$

$$\dot{\mathbf{x}}(t) = \begin{bmatrix} -1 & -1 & 2 & 1 \\ \circ & -1 & \circ & \circ \\ \circ & \circ & -2 & -1 \\ \circ & \circ & 1 & \circ \end{bmatrix} \mathbf{x}(t) + \begin{bmatrix} 1 & -1 \\ 1 & \circ \\ \circ & 1 \\ \circ & \circ \end{bmatrix} \mathbf{v}(t)$$

$$\mathbf{y}(t) = \begin{bmatrix} \circ & 1 & \circ & \circ \\ \circ & \circ & \circ & 1 \end{bmatrix} \mathbf{x}(t)$$

$$\widehat{\mathrm{D}}_d^{-1}(s) = \begin{bmatrix} \dfrac{1}{s+1} & \circ \\ \circ & \dfrac{1}{(s+1)^2} \end{bmatrix}$$

TWO CASE STUDIES

- **Application of the Feedback Decoupling Method in the Attitude Control of the Rolling Aircraft**, *Jie Guo, Shengjing Tang*, Key Laboratory of Dynamics and Control of Flight Vehicle, Ministry of Education, School of Aerospace Engineering Beijing Institute of Technology Beijing, China, 2012 International Conference on Systems and Informatics (ICSAI 2012)

- **State Feedback Decoupling Control of a Control Moment Gyroscope,** *Bruno A. Angélico, Fernando S. Barbosa, Fabio Y. Toriumi*, J Control Autom Electr Syst (2017) 28:26–35 DOI 10.1007/s40313-016-0277-8

Application of the Feedback Decoupling Method in the Attitude Control of the Rolling Aircraft

- Using the small disturbance assumptions, ignoring the influence of the long period velocity deviator and influence of the gravity's normal component to the angular velocity deviator of the trajectory, the 6-DOF motion equations of rolling aircraft can be linearized as:

$$\left.\begin{aligned}
&\frac{d^2\Delta\vartheta}{dt^2} - a_{22}\frac{d\Delta\vartheta}{dt} - a_{24}\Delta\alpha + a'_{27}\Delta\beta - a'_{28}\frac{d\Delta\psi}{dt} = a_{25}\Delta\delta_z^* \\
&\frac{d^2\Delta\psi}{dt^2} - b_{22}\frac{d\Delta\psi}{dt} - b_{24}\Delta\beta - b'_{27}\Delta\alpha + b'_{28}\frac{d\Delta\vartheta}{dt} = -b_{25}\Delta\delta_y^* \\
&\frac{d\Delta\theta}{dt} - a_{34}\Delta\alpha = a_{35}\Delta\delta_z \\
&\frac{d\Delta\psi_c}{dt} - b_{34}\Delta\beta = -b_{35}\Delta\delta_y \\
&-\Delta\vartheta + \Delta\theta + \Delta\alpha = 0 \\
&-\Delta\psi + \Delta\psi_c + \Delta\beta = 0
\end{aligned}\right\} .(21)$$

Where, $\Delta\delta_z^*$ and $\Delta\delta_y^*$ is the equivalent rudder angle under the coordinate of quasi-body, $\omega_y, \omega_z, \beta, \alpha$ are respectively the pitching angular velocity, yawing angular velocity, sideslip angle and attack angle, and other dynamic coefficient are defined as in [8].

$$\begin{bmatrix} \dot{\omega}_y \\ \dot{\omega}_z \\ \dot{\beta} \\ \dot{\alpha} \end{bmatrix} = \begin{bmatrix} b_{22} & -b_{28} & b_{24} & b_{27} \\ a_{28} & a_{22} & -a_{27} & a_{24} \\ 1 & 0 & -b_{34} & 0 \\ 0 & 1 & 0 & -a_{34} \end{bmatrix} \begin{bmatrix} \omega_y \\ \omega_z \\ \beta \\ \alpha \end{bmatrix} + \begin{bmatrix} -b_{25} & 0 \\ 0 & a_{25} \\ b_{35} & 0 \\ 0 & -a_{35} \end{bmatrix} \begin{bmatrix} \delta_y^* \\ \delta_z^* \end{bmatrix}. \quad (22)$$

In order to simplify the design, the characteristics of the actuator is represented by the first-order model, of which the transfer function under the body coordinate system is

$$G_a(s) = \frac{\delta(s)}{u(s)} = \frac{k_a}{\tau_a s + 1}. \quad (23)$$

Where, k_a and τ_a are the actuator gain and the first order time constant. The state equation under the coordinate of quasi-body can be deduced as the following form:

$$\begin{bmatrix} \dot{\delta}_y^* \\ \dot{\delta}_z^* \end{bmatrix} = \begin{bmatrix} -1/\tau_a & -\omega_x \\ -\omega_x & -1/\tau_a \end{bmatrix} \begin{bmatrix} \delta_y^* \\ \delta_z^* \end{bmatrix} + \begin{bmatrix} k_a/\tau_a & 0 \\ 0 & k_a/\tau_a \end{bmatrix} \begin{bmatrix} u_{yc}^* \\ u_{zc}^* \end{bmatrix}. \quad (24)$$

The purpose of our design is to realize one-to-one control from the rudder angle to the angular motion of the rolling aircraft. The coupling system model including body link and actuator link are expressed in (22) and (24), respectively denoted by $\sum(\boldsymbol{A}_{\mathrm{b}}, \boldsymbol{B}_{\mathrm{b}}, \boldsymbol{C}_{\mathrm{b}})$ and $\sum(\boldsymbol{A}_{\mathrm{a}}, \boldsymbol{B}_{\mathrm{a}}, \boldsymbol{C}_{\mathrm{a}})$. Choose $\boldsymbol{x}_{\mathrm{b}}^{\mathrm{T}} = [\omega_y, \omega_z, \beta_y, \beta_z]$ and $\boldsymbol{x}_{\mathrm{a}}^{\mathrm{T}} = [\delta_y^*, \delta_z^*]$ as state variables, while the input and output variables are respectively assigned to $[u_{yc}^*, u_{zc}^*]$, $[\delta_y^*, \delta_z^*]$ and $[\alpha_y^*, \beta_z^*]$. The augmented system matrix is denoted by $\sum(\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{C})$ and the corresponding coefficient matrices are as follows:

$$\boldsymbol{A} = \begin{bmatrix} \boldsymbol{A}_{\mathrm{b}} & \boldsymbol{B}_{\mathrm{b}}\boldsymbol{C}_{\mathrm{a}} \\ \boldsymbol{0} & \boldsymbol{A}_{\mathrm{a}} \end{bmatrix}, \ \boldsymbol{B} = \begin{bmatrix} \boldsymbol{B}_{\mathrm{b}}\boldsymbol{D}_{\mathrm{a}} \\ \boldsymbol{B}_{\mathrm{a}} \end{bmatrix}, \ \boldsymbol{C} = [\boldsymbol{C}_{\mathrm{b}} \quad \boldsymbol{0}].$$

$$
\boldsymbol{A}=\begin{bmatrix} -7.269 & -9.930 & -1582. & 136.2 & -590.38 & 0 \\ 9.930 & -7.269 & -136.2 & -1582.7 & 0 & 590.38 \\ 1 & 0 & -1.5361 & 0 & 0.0786 & 0 \\ 0 & 1 & 0 & -1.536 & 0 & -0.0786 \\ 0 & 0 & 0 & 0 & -100 & -60 \\ 0 & 0 & 0 & 0 & 60 & -100 \end{bmatrix}
\qquad
\boldsymbol{B}=\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 100 & 0 \\ 0 & 100 \end{bmatrix},\ \boldsymbol{C}=\begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}.
$$

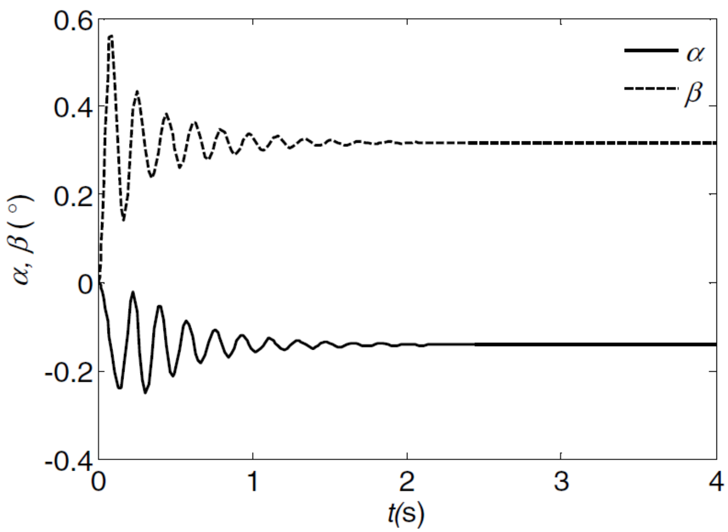


Figure 2. Step response of original system ($u^*_{yc}=0, u^*_{zc}=1$)

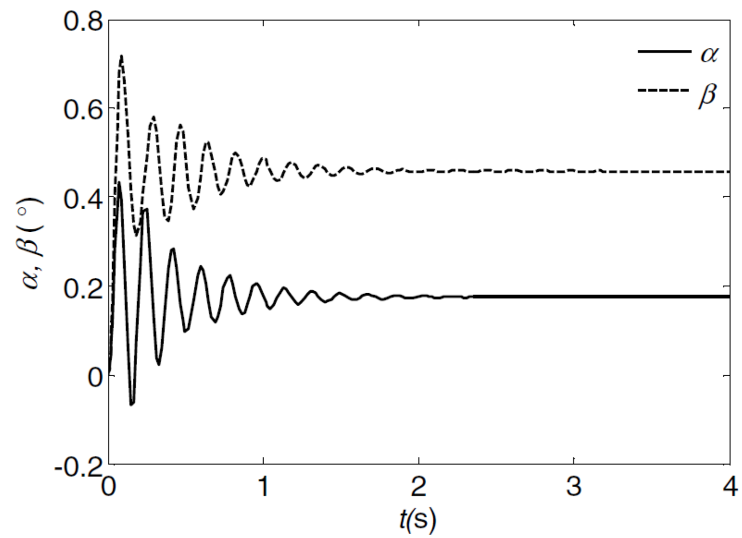


Figure 3. Step response of original system ($u^*_{yc}=1, u^*_{zc}=1$)

$$\Rightarrow d_1 = 1, d_2 = 2$$

$$\Rightarrow (B^*)^{-1} = \begin{bmatrix} 0.1273 & 0 \\ 0 & -0.1273 \end{bmatrix}, \text{ and}$$

$$(B^*)^{-1}\mathrm{B} = (B^*)^{-1}\begin{bmatrix} c_1^T A \\ c_2^T A^2 \end{bmatrix} = \begin{bmatrix} -1.120 & -1.263 & -201.1 & 17.33 & -76.14 & -0.602 \\ -1.263 & 1.120 & 17.33 & 201.1 & 0.602 & -76.141 \end{bmatrix}$$

$$\Rightarrow u = \left(B^*\right)^{-1}\left[-\mathrm{B}x + v\right]$$

And the compensated plant is:

$$G(s) = \begin{bmatrix} \frac{1}{s^2} & 0 \\ 0 & \frac{1}{s^2} \end{bmatrix} \qquad \boldsymbol{G}(s) = 64\begin{bmatrix} \frac{1}{s^2 + 12.8s + 64} & 0 \\ 0 & \frac{1}{s^2 + 12.8s + 64} \end{bmatrix}$$

$$\Rightarrow \hat{C} = \begin{bmatrix} 3.994 & -9.930 & -1536.0 & 136.20 & -597.36 & -4.715 \\ 9.930 & 3.994 & -136.20 & -1536.0 & -4.715 & 597.36 \end{bmatrix}$$

- پاسخ‌های سیستم حلقه‌بسته پس از جبران‌سازی بهره پیش خور:

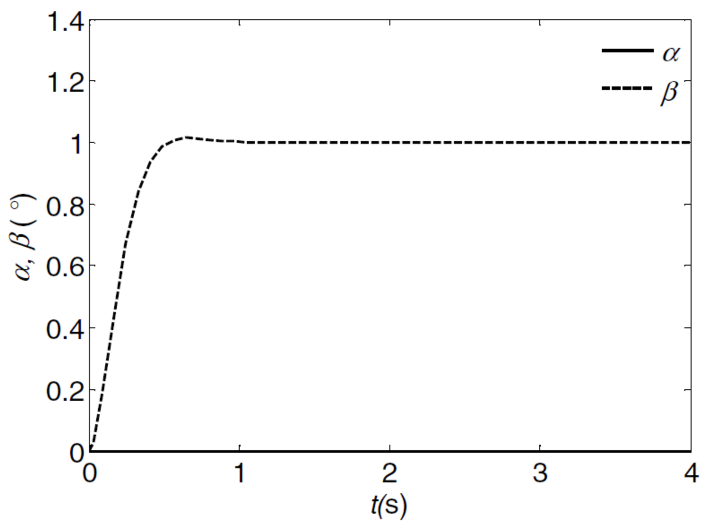


Figure 4. Step response of decoupling system ($u_{yc}^* = 0, u_{zc}^* = 1$)

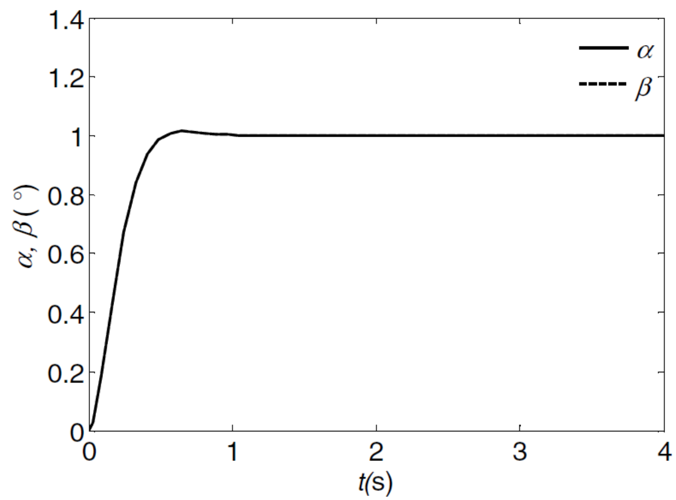


Figure 5. Step response of decoupling system ($u_{yc}^* = 1, u_{zc}^* = 1$)

State Feedback Decoupling Control of a Control Moment Gyroscope

- Control moment gyroscope (CMG) is an actuator commonly used in attitude control of satellites and spacecrafts, as well as in stabilization of marine vessels and unmanned vehicles.
- CMG is a nonlinear multivariable system and presents considerable coupling depending on the chosen operating point, i.e., the gyroscope gimbals angles.
- CMG consists of a spinning wheel (rotor) with motorized gimbals that changes the wheel's angular momentum, causing a gyroscopic torque that rotates the body in space.
- CMGs can produce high torques and are preferred in precision pointing and in handling huge quantities of momentum in large long-duration spacecrafts

- Application of a feedback decoupling control applied to the control moment gyroscope from ECP®:

Fig. 1 CMG eletromechanical plant—ECP model 750

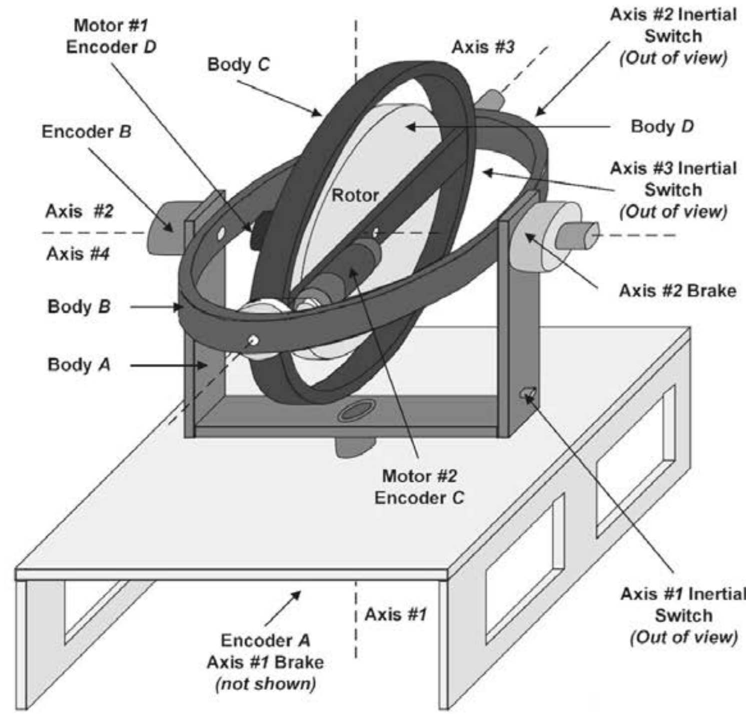


Fig. 2 Components identification of CMG plant

A state space representation is obtained by considering the input, output and state vectors, respectively, defined as $\mathbf{u}(t) = \left[T_1(t)\ T_2(t)\right]^\top$, $\mathbf{y}(t) = \left[\theta_2(t)\ \theta_1(t)\right]^\top$ and $\mathbf{x}(t) = \left[\theta_2(t)\ \theta_1(t)\ \omega_3(t)\ \omega_2(t)\ \omega_1(t)\right]^\top$, with

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 2.7039 & -21.9216 & 58.0068 \\ 0 & 0 & 5.3269 & 0 & 2.7776 \\ 0 & 0 & -5.1857 & 0 & -2.7039 \end{bmatrix}, \tag{11}$$

$$\mathbf{B} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 2.5799 & 418.5058 \\ -35.6283 & -0.7057 \\ -4.9479 & -19.7751 \end{bmatrix}, \tag{12}$$

$$\mathbf{C} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}, \tag{13}$$

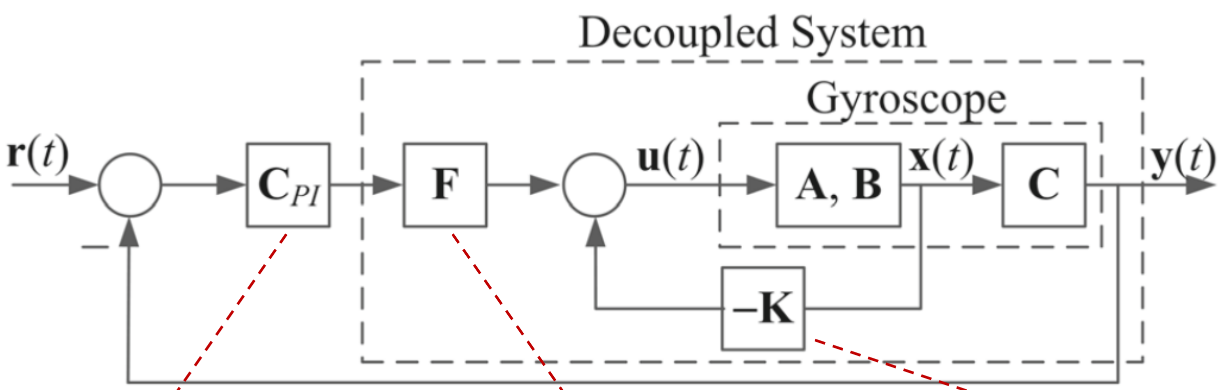
the state, input and output matrices. One can note that the state θ_3 was eliminated, since it is not controllable with only two actuators and the other states do not depend on it. It is a controllable, observable and minimum phase system with a transmission zero located at $s = -122.92$. The equivalent transfer function matrix is:

$$\mathbf{G}(s) = \begin{bmatrix} \frac{-35.63s^2-11950}{s^4+410.3s^2} & \frac{-0.7057s^2+2174s-6169}{s^4+410.3s^2} \\ \frac{-4.948s^2-4628}{s^4+410.3s^2} & \frac{-19.78s^2-2117s-2389}{s^4+410.3s^2} \end{bmatrix} \quad (14)$$

$$\Rightarrow d_1 = 2, d_2 = 2 \qquad \left| B^* \right| \neq 0$$

$$\mathbf{G_d}(s) = \begin{bmatrix} \frac{0.7}{s^2+7.1s+0.7} & 0 \\ 0 & \frac{0.7}{s^2+7.1s+0.7} \end{bmatrix}$$

- سیستم حلقه‌بسته با فیدبک حالت دکوپله‌ساز و جبران‌ساز قطری:



$$\mathbf{K} = \begin{bmatrix} -0.0197 & 0.0007 & -0.1555 & -0.2003 & -0.0739 \\ 0.0049 & -0.0356 & 0.3011 & 0.0501 & -0.2038 \end{bmatrix}$$

$$\mathbf{C}_{PI}(s) = \begin{bmatrix} 29.2571\left(1+\frac{1}{0.7847s}\right) & 0 \\ 0 & 29.2571\left(1+\frac{1}{0.7847s}\right) \end{bmatrix}$$

$$\mathbf{F} = \begin{bmatrix} -0.0197 & 0.0007 \\ 0.0049 & -0.0356 \end{bmatrix}$$

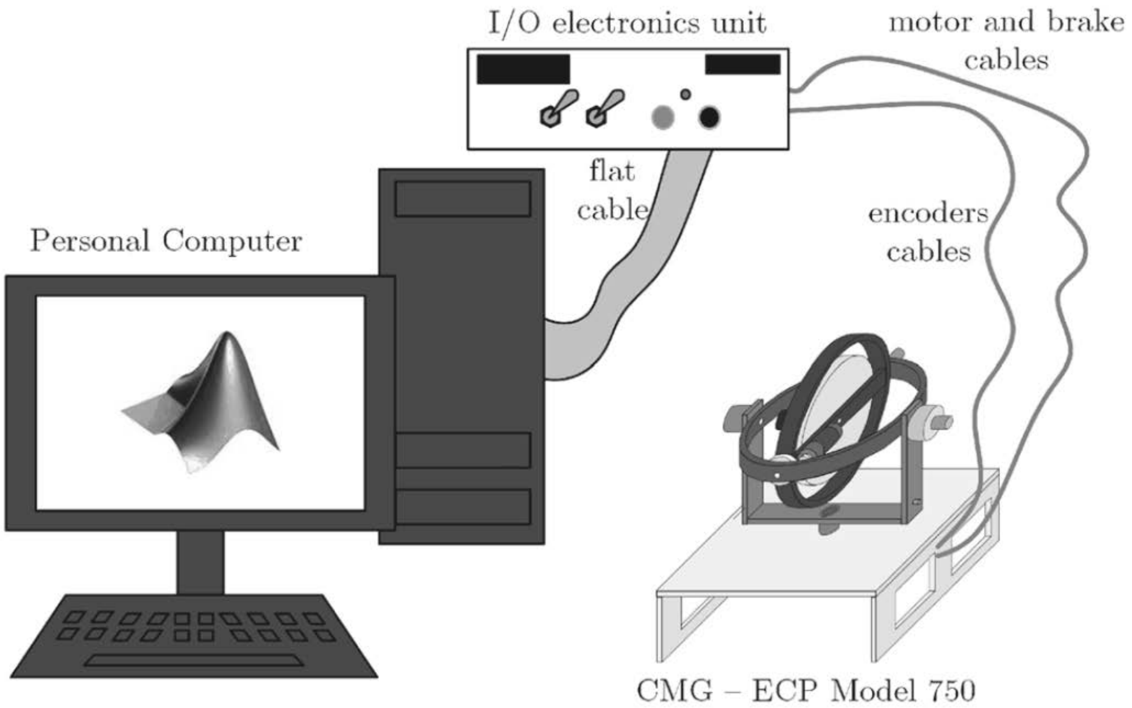


Fig. 5 Experimental setup

- The experimental setup is composed of three subsystems: personal computer running MATLAB, input/output electron-ics unit and the CMG electromechanical plant. Input/output (I/O) electronics unit is composed of servo amplifiers and brake commands. Furthermore, there is a digital signal processor (DSP)-based real-time controller installed in the personal computer that communicates with the I/O electron-ics unit by flat cable.

Before the multivariable control starts, the system is taken to its operation point. Gimbal 2 and 3 angles are positioned such that $\theta 2 = -20^{\circ}$ and $\theta 3 = 20^{\circ}$, and brakes of axes 1 and 2 are turned on. Hence, an initial control loop with a PI speed controller acts in motor M#1 in order to speed the rotor disk (flywheel) to 400 rpm, resulting in a greater control effort on this motor in the first seconds. At 13 seconds, breaks are released, since the system is in equilibrium. At 15 seconds, the multivariable control algorithm starts.

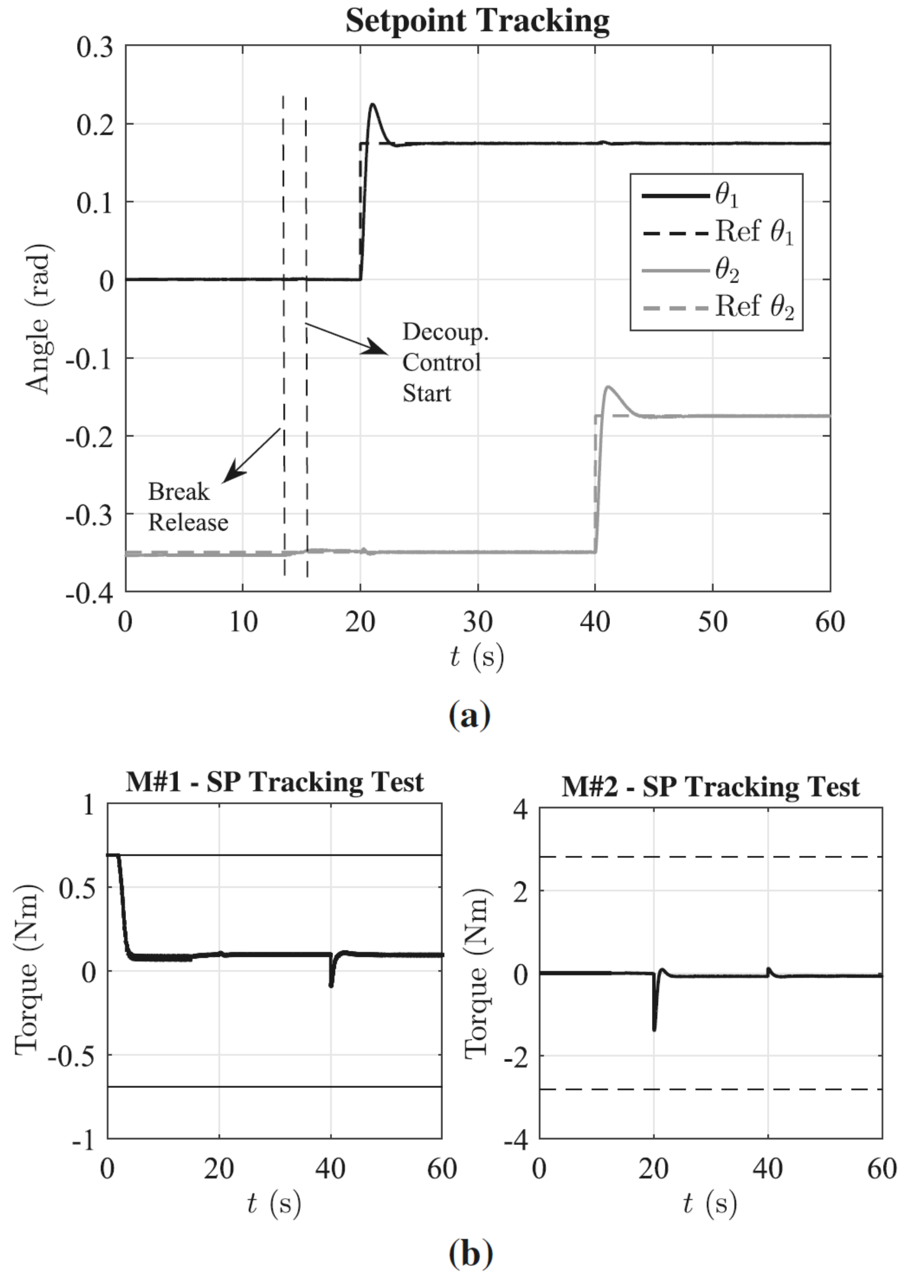


Fig. 10 Experimental results considering positive step response for θ_1 and θ_2: **a** setpoint of $+10°$ around the equilibrium point, **b** control effort

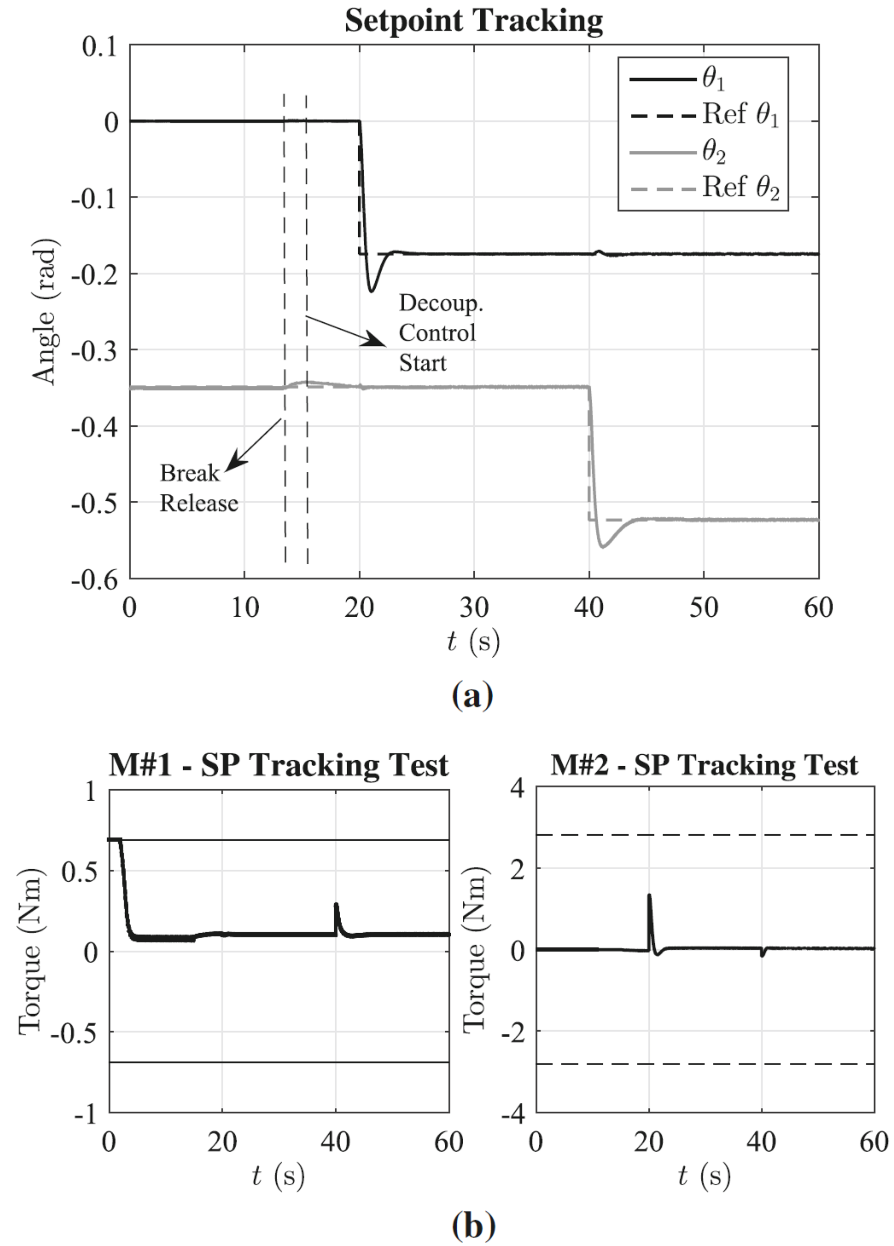


Fig. 11 Experimental results considering negative step response for θ_1 and θ_2: **a** setpoint of $-10°$ around the equilibrium point, **b** control effort

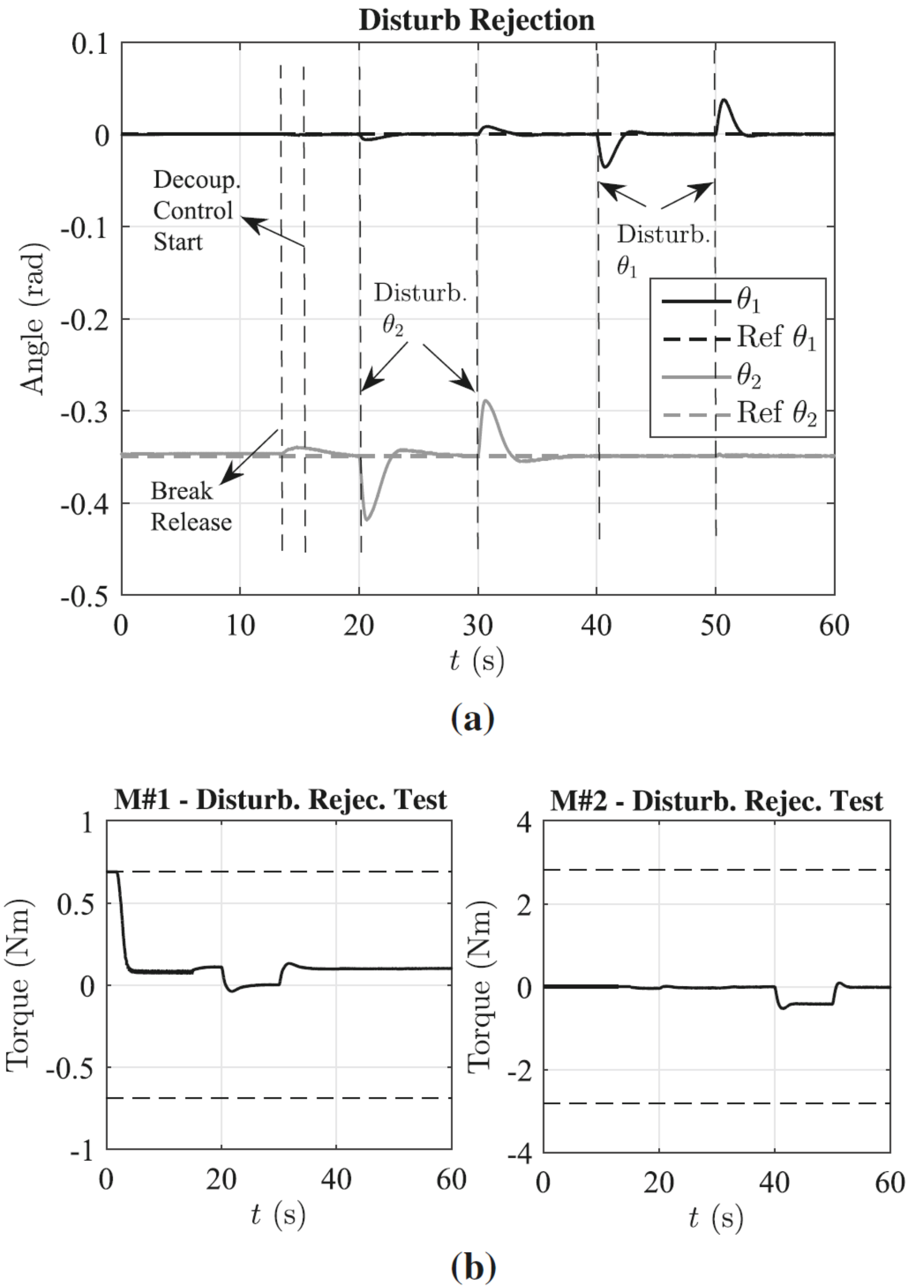


Fig. 12 Experimental results for disturbance applied to the inputs of θ_1 and θ_2: **a** positive torque disturbance, **b** control effort